Name: Key Date: ________ Class Period: ______

6.3 – Logarithms

Solve each equation and simplify your answer to an integer or fraction. Be sure to show your method. ***NC***

1. $2\log_3(x+2)-5=1$

$2\log_3(x+2)=6$

$\log_3(x+2)=3$

$3^3=x+2$

$\boxed{x=25}$

2. $3\cdot 10^{2x+5}+4=3{,}004$

$3\cdot 10^{2x+5}=3000$

$10^{2x+5}=1000$

$\log 1000=2x+5$

$2x=-2$

$\boxed{x=-1}$

3. $16^{4x-1}=4^{4x+2}$

$(4^2)^{4x-1}=4^{4x+2}$

$4^{8x-2}=4^{4x+2}$

$8x-2=4x+2$

$4x=4$

$\boxed{x=1}$

4. $\log_x 49+4\log_2 32=22$

$\log_x 49+4(5)=22$

$\log_x 49=2$

$x^2=49$

$\boxed{x=7}$ $x=-7$ extraneous!

(base can't be negative)

5. $\left(\frac{3}{2}\right)^{2x}=\left(\frac{16}{81}\right)^{x-3}$

$\left(\frac{3}{2}\right)^{2x}=\left(\left(\frac{2}{3}\right)^4\right)^{x-3}$

$\left(\frac{3}{2}\right)^{2x}=\left(\frac{3}{2}\right)^{-4(x-3)}$

$2x=-4x+12$

$6x=12$

$\boxed{x=2}$

6. $\log_{25}5=4\cdot 2^{x-5}$

$\frac{1}{2}=4\cdot 2^{x-5}$

$\frac{1}{8}=2^{x-5}$

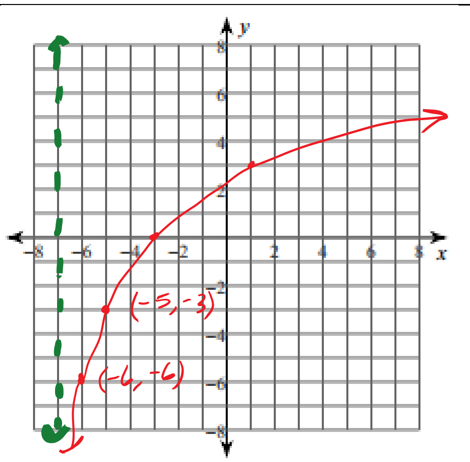
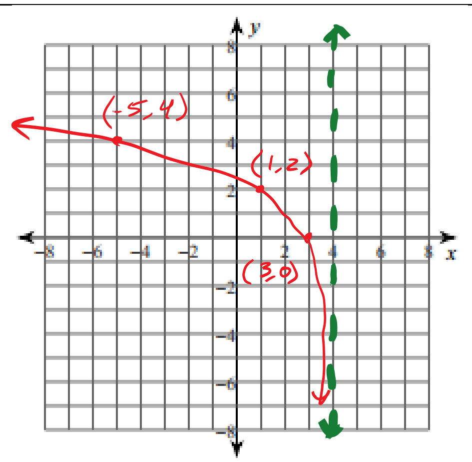
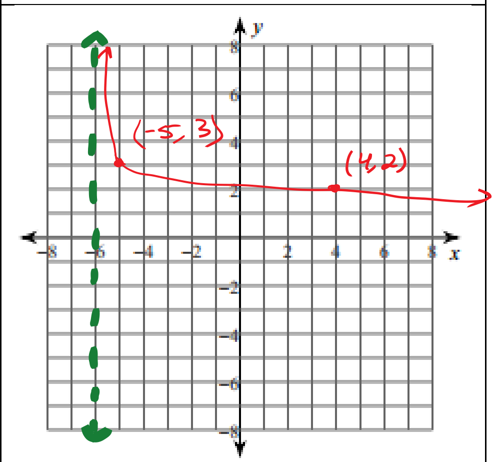
$2^{-3}=2^{x-5}$

$-3=x-5$

$\boxed{x=2}$

6.4 – Logarithmic Functions

Graph each function, then answer the questions below. Note: AROC = Average Rate of Change. ***NC***

7. $f(x)=3\log_2(x+7)-6$	**8.** $g(x)=2\log_3(-(x-4))$	**9.** $h(x)=-\log(x+6)+3$
		
Transformations: vertical dilation by s.f. of 3; horizontal translation left 7 and vertical translation down 6	**Transformations:** Reflection across y-axis, vertical dilation by s.f. of 2, horizontal translation right 4	**Transformations:** Reflection across x-axis, vertical translation up 3, horizontal translation left 6
Domain: $x\in(-7,\infty)$	**x-intercept:** $(3,0)$	**Domain:** $x\in(-6,\infty)$
Range: $f(x)\in\mathbb{R}$	**AROC on** $x\in[-5,1]$**:** $-\frac{1}{3}$	**AROC on** $x\in[-5,4]$**:** $-\frac{1}{9}$
x-intervals where $f(x)<0$**:** $x\in(-7,-3)$	**End Behavior:** as $x\to 4^-$, $g(x)\to-\infty$; as $x\to-\infty$, $g(x)\to\infty$	**Asymptote:** $x=-6$

10. Find the inverse of the functions below. Be sure to show work and use correct labels! ***NC***

a. $f(x) = -2\log_4(x-4)+3$

$y = -2\log_4(x-4)+3$

$\frac{y-3}{-2} = \log_4(x-4)$

$4^{-\left(\frac{y-3}{2}\right)} = x-4$

$x = 4^{-\left(\frac{y-3}{2}\right)}+4$

$\boxed{\therefore f^{-1}(x) = 4^{-\left(\frac{x-3}{2}\right)}+4}$

b. $g(x) = 3^{(x-2)}+1$

$y = 3^{x-2}+1$

$y-1 = 3^{x-2}$

$\log_3(y-1) = x-2$

$x = 2+\log_3(y-1)$

$\boxed{\therefore g^{-1}(x) = 2+\log_3(x-1)}$

c. $h(x) = \log_2(-(x+4))-10$

$y = \log_2(-(x+4))-10$

$y+10 = \log_2(-(x+4))$

$2^{y+10} = -(x+4)$

$-2^{y+10} = x+4$

$x = -2^{y+10}-4$

$\boxed{\therefore h^{-1}(x) = -2^{x+10}-4}$

6.7 – Geometric Sequences and Series

For each series below, first write the explicit formula (a), write in sigma notation (b), then find the sum (c). If the sum doesn't exist, explain why. ***NC***

11. $2-6+18+...+162$

a. $a_1 = 2$; $r = -3$

$\boxed{\therefore a_n = 2(-3)^{n-1},\ n \geq 1}$

b. $162 = 2(-3)^{n-1} \rightarrow (-3)^{n-1} = 81$

$\therefore n = 5$

$\boxed{\therefore \sum_{n=1}^{5} 2(-3)^{n-1}}$

c. $S_5 = \frac{2(1-(-3)^5)}{1--3} = \frac{2(244)}{4}$

$\boxed{S_5 = 122}$

12. $\frac{1}{3}+\frac{5}{9}+\frac{25}{27}+...$

a. $a_1 = \frac{1}{3}$; $r = \frac{5}{3}$

$\boxed{\therefore a_n = \frac{1}{3}\left(\frac{5}{3}\right)^{n-1},\ n \geq 1}$

b. $\boxed{\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{5}{3}\right)^{n-1}}$

c. $\boxed{\text{DNE since } r = \frac{5}{3} \text{ and } \left|\frac{5}{3}\right| > 1.}$

13. $44-11+\frac{11}{4}-\frac{11}{16}+...$

a. $a_1 = 44$; $r = -\frac{1}{4}$

$\boxed{\therefore a_n = 44\left(-\frac{1}{4}\right)^{n-1},\ n \geq 1}$

b. $\boxed{\sum_{n=1}^{\infty} 44\left(-\frac{1}{4}\right)^{n-1}}$

c. $S = \frac{44}{1--\frac{1}{4}} = 44 \div \frac{5}{4} = 44 \cdot \frac{4}{5}$

$\boxed{\therefore S = \frac{176}{5}}$

14. *Fill in each blank with "converges" or "diverges."* Question (12) is an example of a series that diverges, and question (13) is an example of a series that converges.

15. Evaluate $\sum_{n=1}^{13} 3.2(4.1)^{n-1}$. ***C***

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{13} = \frac{3.2(1-4.1^{13})}{1-4.1}$

$\boxed{S_{13} = 95494512.7551}$

16. If the first term of a series is 3, the common ratio is 5, and $S_k = 292{,}968$, solve for *k*. ***C***

$S_n = \frac{a_1(1-r^n)}{1-r}$

$292{,}968 = \frac{3(1-5^k)}{1-5}$

$\left(-\frac{4}{3}\right)292{,}968 = 1-5^k$

$-5^k = -390{,}624-1$

$k = \log_5 390{,}625$

$\boxed{k=8}$ (8 terms were added)

Application Problems

17. Shari bought a TV for \$1,798. The value of this TV depreciates exponentially by 8% each year. ***C***

a. Write an exponential function $f(x) = a \cdot b^x$ that models the TV's value, $f(x)$, after x years since it was purchased.

$a = 1798$; $b = 1 - r \rightarrow b = 0.92$ $\quad \boxed{\therefore f(x) = 1798(0.92)^x}$

b. To the nearest year since it was purchased, when is the TV's value \$500.00?

$f(x) = 500$

$\therefore 500 = 1798(0.92)^x$

$(0.92)^x = \dfrac{500}{1798} \rightarrow \log_{(0.92)}\left(\dfrac{500}{1798}\right) = x$

$x \approx 15.34897\ldots$

∴ In the 15th year since purchasing the TV, its value is \$500.

18. The decibel (dB) scale is based on the formula below. A typical vacuum cleaner registers at 80 dB, and human breathing registers at 10 dB. How many times more intense in sound is a typical vacuum cleaner compared to human breathing? ***NC***

$N_{dB} = 10\log\left(\dfrac{I}{I_0}\right)$; N_{dB} is the change in decibel level; $\dfrac{I}{I_0}$ is the intensity ratio of the two sounds

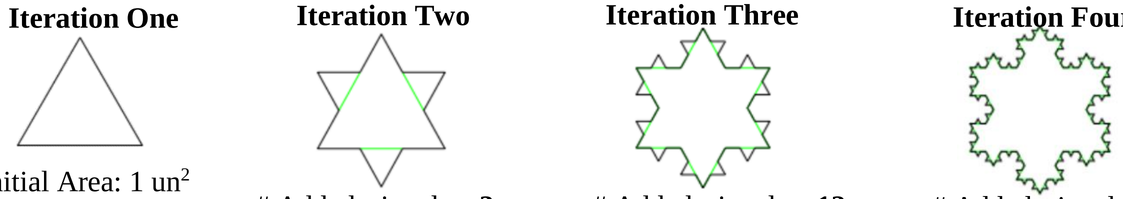
$N_{dB} = 80 - 10 = 70$

$\therefore 70 = 10\log\left(\dfrac{I}{I_0}\right) \rightarrow 7 = \log\left(\dfrac{I}{I_0}\right)$

$10^7 = \dfrac{I}{I_0} \rightarrow \dfrac{I}{I_0} = 10{,}000{,}000$

∴ A vacuum cleaner is 10,000,000 times more intense in sound.

19. The Koch Snowflake is a fractal – a self-similar figure where a part has the same characteristics as the whole. Geometric properties of fractals, such as perimeter and area, can be represented using geometric sequences and series. To create the Koch Snowflake, the sides of an equilateral triangle are divided into thirds, and a smaller equilateral triangle is added onto each side. The side lengths of the added triangles are one-third the size of the previous triangle. A few iterations of the Koch Snowflake are shown below.



Iteration One	Iteration Two	Iteration Three	Iteration Four
Initial Area: 1 un^2	# Added triangles: **3** Added Area: **?**	# Added triangles: **12** Added Area: **?**	# Added triangles: **48** Added Area: **?**

a. If the scale factor of the sides of each added triangle is 1/3, what will be the scale factor of the area? *Hint: Generalize! How does the area of a 4" x 4" square compare to the area of a 12" x 12" square?*

$\left(\dfrac{1}{3}\right)^2 = \boxed{\dfrac{1}{9}}$ ∴ The area of each added triangle is $\frac{1}{9}$ the area of the previous triangle.

b. The geometric series below expresses the added area starting at iteration two. Using your answer from (a), fill in the blanks. $3\left(\dfrac{1}{9}\right) + 12\left(\dfrac{1}{9}\right)^2 + 48\left(\dfrac{1}{9}\right)^3 + \ldots$

c. Based on (b), how much area will be added? What will be the total area in the Koch Snowflake?

$\dfrac{3}{9} + \dfrac{12}{81} + \dfrac{48}{729} + \ldots$

$\times \dfrac{4}{9}$ $\quad \times \dfrac{4}{9}$

$a_1 = \dfrac{3}{9}$ or $\dfrac{1}{3}$; $r = \dfrac{4}{9}$

$\therefore S = \dfrac{\frac{1}{3}}{1 - \frac{4}{9}} = \dfrac{1}{3} \cdot \dfrac{9}{5} = \dfrac{3}{5}$

∴ In part (b), the total added area is $\frac{3}{5}$.

∴ The total area in the Koch Snowflake is $\frac{8}{5}$ $\left(1 + \frac{3}{5}\right)$.