1.4-1.7 Guided Notes

1.4 Inductive Reasoning

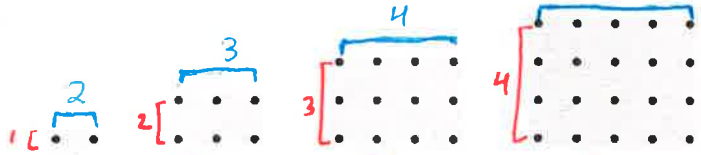
Description	Example
Sequence: an ordered list of numbers.	2, 4, 6, 8, ... 20, 10, 5, 2.5, ...
Conjecture: an unproven statement or rule based on patterns and observations	If Mrs. Leon is drinking out of a mug, she is drinking tea.
Counterexample: an example that shows a conjecture is false.	From above, Mrs. Leon might be drinking coffee.
Inductive Reasoning: a type of reasoning that reaches conclusions based on a pattern of specific examples or past events.	Patterns apply to shapes, numbers, or behaviors.

Example 1: Use inductive reasoning to find the next two terms of the sequence.

88, 82, 76, 70, 64, ... (−6, −6, −6, −6)

∴ 58 and 52 are the next numbers.

Example 2: Use inductive reasoning to make a conjecture about the number of dots in the nth term of the geometric pattern to the right.

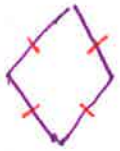


Term	1	2	3	4
Number of Dots	2	6	12	20
Numeric Relationship:	(1)(2)	(2)(3)	(3)(4)	(4)(5)
Terms of n:	$(n)(n+1)$	$(n)(n+1)$	$(n)(n+1)$	$(n)(n+1)$

Example 3: Test each conjecture below by finding several true examples or a single counterexample.

False

Conjecture: A polygon with four congruent sides is a square.

Counterexample:  Diamond!

False

Conjecture: The square of a number is larger than the original number.

Test: 2, 3, 4:

$2^2 > 2$ $3^2 > 3$ $4^2 > 4$

Test: 0, 1

$0^2 \not> 0$ False! $1^2 \not> 1$ False!

Counterexample: 0 and 1 are counterexamples.

True? (need proof)

Conjecture: The difference of two consecutive perfect squares is always an odd number. Others???

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, ...

Test: 1, 4, 9, 16, 25, 36, 49 ...

3, 5, 7, 9, 11, 13

Seems true.

1.4-1.7 Guided Notes

1.5-1.6 Conditional Statements and Deductive Reasoning

If-Then Statement: Statements that show cause and effect. These statements can be true or false.

Cause	Effect	
If it is raining 	then it is spring. 	False! Could be summer.
If x and y are whole numbers	then their difference $x - y$ is a whole number. → 0, 1, 2, 3, ...	False! 1-2 is -1, an integer.
If water is heated 	then it boils. 	False! Needs to be to certain temp.
If a triangle has a right angle	then it is a right triangle.	True!
If your favorite color is blue 	then you are a good speller.	False! Mrs. Karpenko is a counterexample

Conditional Statement: an if-then statement that relates a **hypothesis**, the part that follows "if", to a **conclusion**, the part that follows "then".

Notation: Let p be the hypothesis, and q be the conclusion. $p \rightarrow q$ is read as "If p, then q."

Example 1: Write the statement as a conditional. Keep in mind to write "if *cause*, then *effect*."

You can register to vote if you are at least 18 years old.

Converse: reverses the hypothesis and conclusion in a conditional.

Example 2: Write the converse of each conditional statement below, then determine if it is true. If the converse is false, explain why.

A. If you play the trumpet (p), then you play a brass instrument (q).

Converse: If you play a brass instrument, then you play the trumpet.
False! Counterexample is the trombone.

B. If two angles form a linear pair (p), then they are supplementary (q).

Converse: If two angles are supplementary, then they form a linear pair.
False! Counterexample: two right angles that aren't adjacent.

C. If all three sides in a triangle are congruent (p), then the triangle is equilateral (q).

Converse: If a triangle is equilateral, then all three sides are congruent.

TRUE!!

1.4-1.7 Guided Notes

Deductive Reasoning: a type of reasoning that uses given and previously known facts to reach a logical conclusion.

Example 3: Carly solved the following problem: $3(x-6)+14=35$. Each step of her work is shown below. Use the word bank to right in the justification for each step she made.

Carly's Work	Justification
$3(x-6)+14=35$ $-14 \quad -14$	Original Problem
$\frac{3(x-6)}{3}=\frac{21}{3}$	Subtraction Property of Equality
$x-6=7$ $+6 \quad +6$	Division Property of Equality
$x=13$	Addition Property of Equality

Word Bank

Addition Property of Equality

Subtraction Property of Equality

Multiplication Property of Equality

Division Property of Equality

Distributive Property of Multiplication

1.7 Writing Proofs

Review Vocabulary: Look at the equations below and think about what you would do to solve for the variable. Fill in the letter of the correct Property of Real Numbers that matches what you would do to solve it.

D	$5 = x + 7$	A. Addition Property of Equality
A	$y - 12 = 23$	B. Division Property of Equality
B	$8z = 64$	C. Multiplication Property of Equality
C	$\frac{W}{3} = 48$	D. Subtraction Property of Equality

Example 1: Solve the following equation for x. For each step you take, write the justification for that step next to it. A table has been provided for your convenience.

Your Work	Reason
1. $\frac{2x+5}{3}=9$	1. Given
2. $2x+5=27$	2. Multiplication Prop. of Equality
3. $2x=22$	3. Subtraction Prop. of Equality
4. $x=11$	4. Division Prop. of Equality

1.4-1.7 Guided Notes

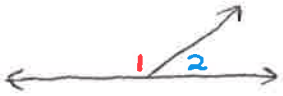
Transitive Property of Equality: If $a = b$ and $b = c$, then $a = c$.

Transitive Property of Congruence: If $A \cong B$ and $B \cong C$, then $A \cong C$.

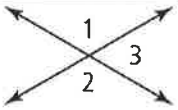
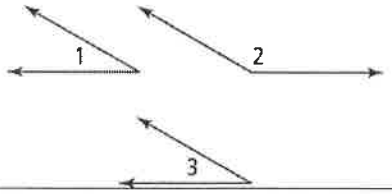
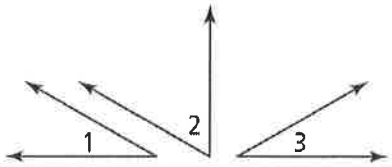
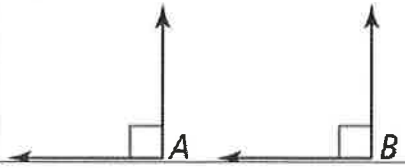
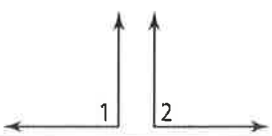
Proof: A convincing argument that uses deductive reasoning most of the time. The format is usually two-column or flow-chart.

Theorem: A conjecture that has been proven to be true.

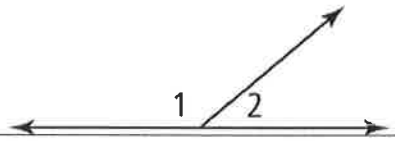
Linear Pair: two angles that form a line. Picture:



Useful Theorems

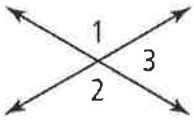
Words	Symbols	
Vertical Angles Thm (1-1): Vertical angles are congruent.	Hypothesis: 	Conclusion: $\angle 1 \cong \angle 2$
Congruent Supplements Thm (1-2): If two angles are supplementary to the same angle (or congruent angles), then the two angles are congruent.	Hypothesis: $m\angle 1 + m\angle 2 = 180°$ and $m\angle 3 + m\angle 2 = 180°$ 	Conclusion: $\angle 1 \cong \angle 3$
Congruent Complements Thm (1-3): If two angles are complementary to the same angle (or congruent angles), then the two angles are congruent.	Hypothesis: $m\angle 1 + m\angle 2 = 90°$ and $m\angle 3 + m\angle 2 = 90°$ 	Conclusion: $\angle 1 \cong \angle 3$
Right Angles Thm (1-4): All right angles are congruent.	Hypothesis: 	Conclusion: $\angle A \cong \angle B$
Congruent Supplements Thm (1-5): If two angles are congruent and supplementary, then each is a right angle.	Hypothesis: $\angle 1 \cong \angle 2$ and $m\angle 1 + m\angle 2 = 180°$ 	Conclusion: $m\angle 1 = 90°$ $m\angle 2 = 90°$

1.4-1.7 Guided Notes

Linear Pairs Thm (1-6): The sum of the measures of a linear pair is 180°.	**Hypothesis:** 	**Conclusion:** $m\angle 1 + m\angle 2 = 180°$

Example 2: Fill in the two-column proof of the Vertical Angles Theorem below.

Given: ∠1 and ∠2 are vertical angles
Prove: ∠1 ≅ ∠2

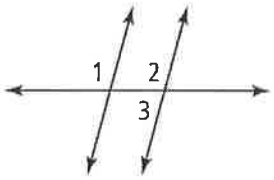


Statements	Reasons
1) ∠1 and ∠2 are vertical angles	1) Given
2) $m\angle 2 + m\angle 3 = 180°$ $m\angle 1 + m\angle 3 = 180°$	2) Supplementary Angles
3) $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	3) Transitive Property of Equality
4) $m\angle 1 = m\angle 2$	4) Subtraction Property of Equality
5) ∠1 ≅ ∠2	5) Definition of Congruent Angles

Example 3: Fill in the two-column proof below.

Given: $m\angle 1 = m\angle 2$, $m\angle 1 = 105°$

Prove: $m\angle 3 = 75°$



Statements	Reasons
1) $m\angle 1 = m\angle 2$	1) Given
2) $m\angle 1 = 105°$	2) Given
3) $m\angle 2 = 105°$	3) Substitution or Transitive Prop. of =
4) ∠2 and ∠3 are a linear pair	4) Definition of a linear pair
5) $m\angle 2 + m\angle 3 = 180°$	5) Linear Pairs Theorem
6) $105° + m\angle 3 = 180°$	6) Substitution Property of Equality
7) $m\angle 3 = 75°$	7) Subtraction Property of Equality