KEY

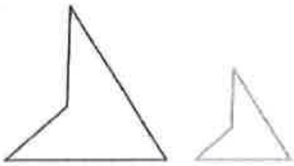
3.1-3.5 Guided Notes

3.1 Reflections

Rigid Motion: transformation that preserves length and angle measure.

Are the transformations rigid motion?

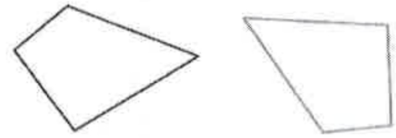
a)



preimage   image

No, the size changes

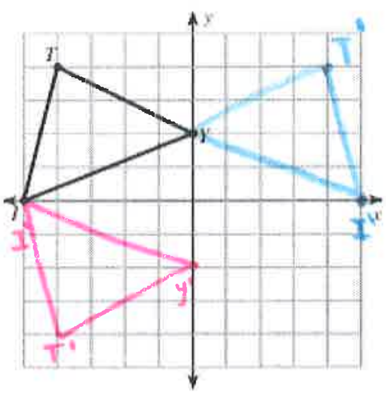
b)



preimage   image

yes

Reflection: a transformation that reflects each point across a line of reflection.



1) $R_{x\text{-axis}}$ State the coordinates of the new shape.

$(x, y) \rightarrow (x, -y)$

$(-5, 0), (0, -2), (-4, -4)$

2) $R_{y\text{-axis}}$ State the coordinates of the new shape.

$(x, y) \rightarrow (-x, y)$

$(0, 2), (5, 0), (4, 4)$

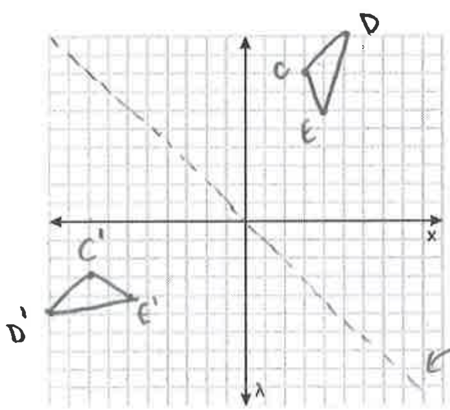
Reflection across the x-axis: $(x, -y)$

Reflection across the y – axis: $(-x, y)$

Example 1:

Draw the triangle on the coordinate plane. Then sketch it's image.

a) C(3, 8), D(5, 10), E(4, 6)
C'(-8,-3), D'(-10, -5), E'(-6,-4)



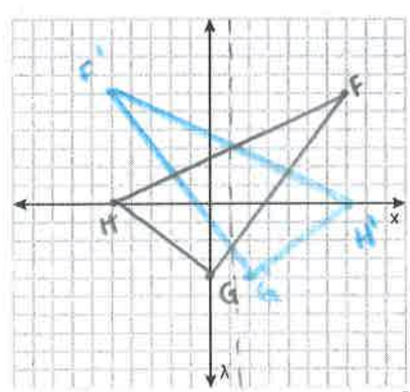
Line of Reflection

Rule:

$(-y, -x)$

$R_{y=-x}$

b) F (7, 6), G(0,-4), H(-5,0)
F'(-5, 6), G'(2, -4), H'(7, 0)



Line of Reflection

Rule:

$R_{x=1}$

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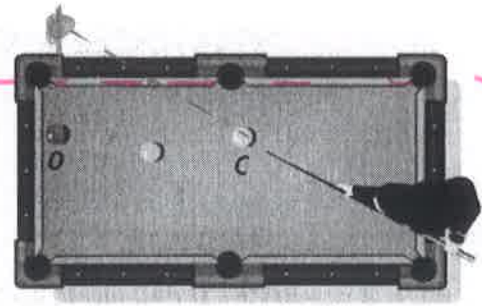
Example 2:

In a billiards game, a player must hit the cue ball(C) so that the cue ball hits ball (O) without touching the other ball. Where should the cue ball bounce off the top rail so that it hits ball (O)?

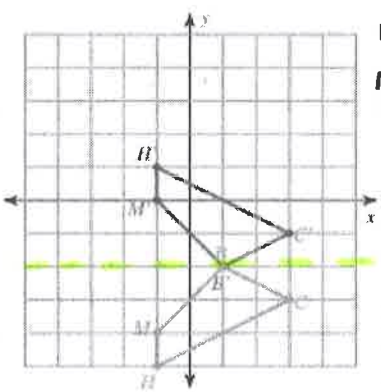


1) The top rail is the line of reflection, where the ball will bounce.
2) Draw an image of ball (O) so that it is perpendicular to the line of reflection and equidistant.
3) Draw a segment from the cue ball to the image of the ball.

You try!

Determine the line of reflection and the rule of reflection.

a)



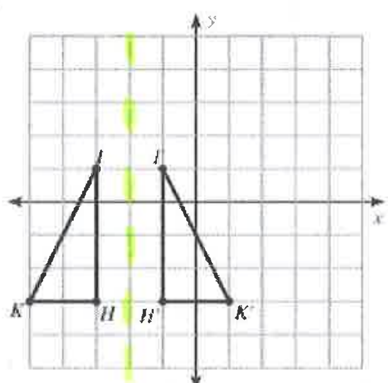
H(-1,-5)
M(-1,-4)
E(1,-2)
C(3,-3)

y = -2

H' (-1,1)
M' (-1,0)
E' (1,-2)
C' (3,-1)

(x, -y-4)

b)



x = -2

K (-5,-3)
I (-3,1)
H (-3,-3)

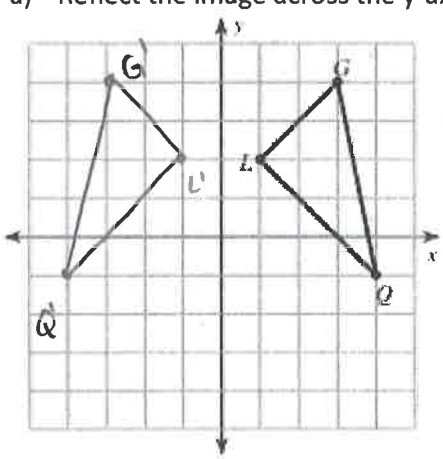
K' (1,-3)
I' (-1,1)
H' (-1,-3)

(-x-4, y)

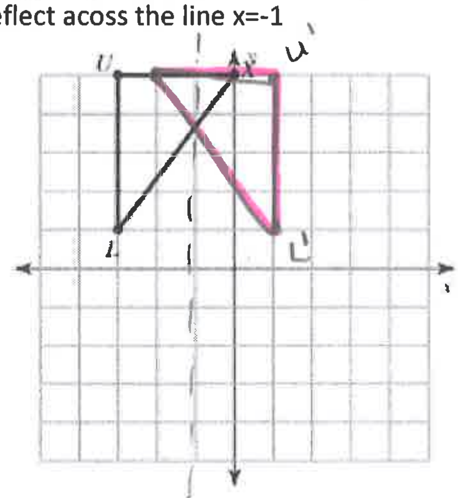
3.2 Translations

Warm-Up

a) Reflect the image across the y-axis.



b) Reflect acoss the line x=-1



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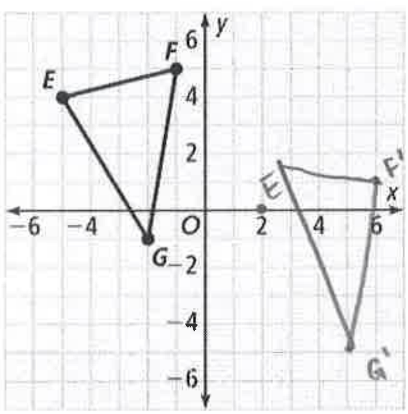
Translations: a translation is a transformation in a plane that maps all points of a preimage the same distance and in the same direction.

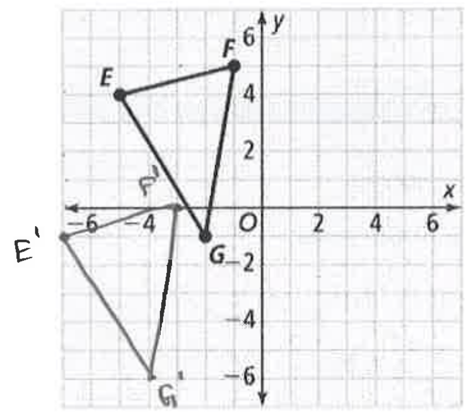
Example 1: Perform the indicated translation.

a) Graph $T_{\langle 7,-4\rangle} = \Delta E'F'G'$?



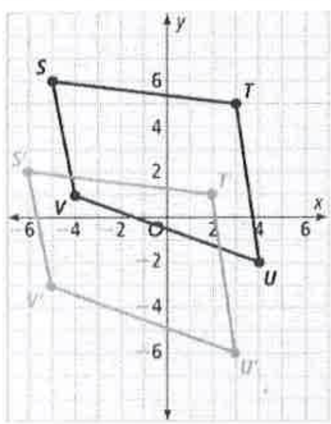
translate right 7 & down 4

b) Graph $T_{\langle -2,-5\rangle} = \Delta E'F'G'$?



translate left 2, down 5

Example 2:
What translation rule maps the preimage onto the image?

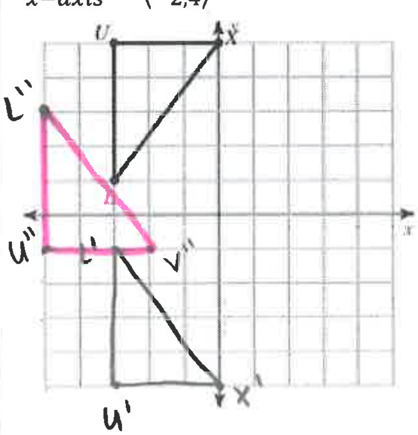


translated left 1 & down 4

$T_{\langle -1,-4\rangle}$

Composition of Rigid Motion: transformation with two or more rigid motions in which the second rigid motion is performed on the image of the first rigid motion.

$R_{x-axis} \cdot T_{\langle -2,4\rangle}$



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Example 3:
Write the composition of the transformations as a single transformation.

a) $T_{\langle 3,-2\rangle} \cdot T_{\langle 1,3\rangle}$

$T_{\langle 3+1,\,-2+3\rangle} = T_{\langle 4,1\rangle}$

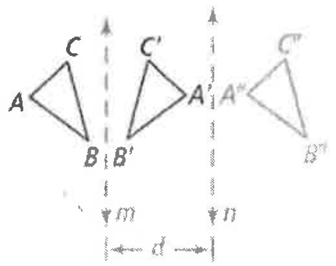
b) $T_{\langle -4,0\rangle} \cdot T_{\langle -2,5\rangle}$

$T_{\langle -4+-2,\,0+5\rangle} = T_{\langle -6,5\rangle}$

A composition of reflections across two parallel lines is the same as a translation

If... $T(ABC) = A''B''C''$

$AA'' = BB'' = CC'' = 2d$



Then... $(R_n \circ R_m)(ABC) = A''B''C''$

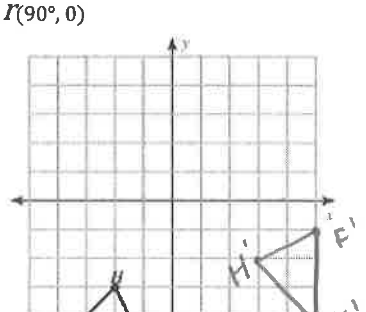
3.3 Rotations

Rotation is a transformation that rotates each point in the preimage about a point P, called the center of rotation, by an angle measure of x°, called the angle of rotation.

* Counterclockwise for positive angle measure

Copy ΔUHF on patty paper. Then perform the indicated operation.

1. $r_{(90°,\,0)}$

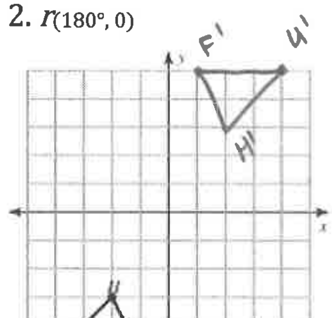


Rule: $(x, y) \rightarrow (-y, x)$

	x	y
H	-2	-3
U	-4	-5
F	-1	-5

x	y
3	-2
5	-4
5	-1

2. $r_{(180°,\,0)}$

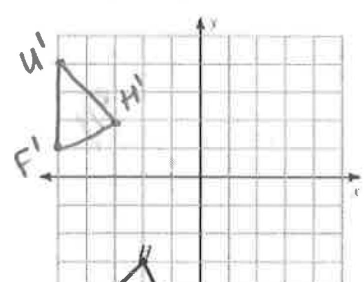


Rule: $(x, y) \rightarrow (-x, -y)$

	x	y
H	-2	-3
U	-4	-5
F	-1	-5

x	y
2	3
4	5
1	5

3. $r_{(270°,\,0)}$



Rule: $(x, y) \rightarrow (y, -x)$

	x	y
H	-2	-3
U	-4	-5
F	-1	-5

x	y
-3	2
-5	4
-5	1

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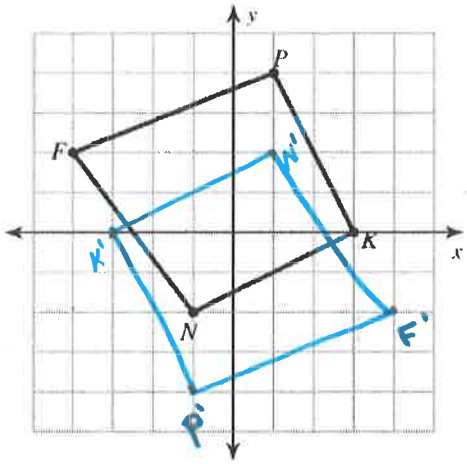
Additional Practice with Rotations

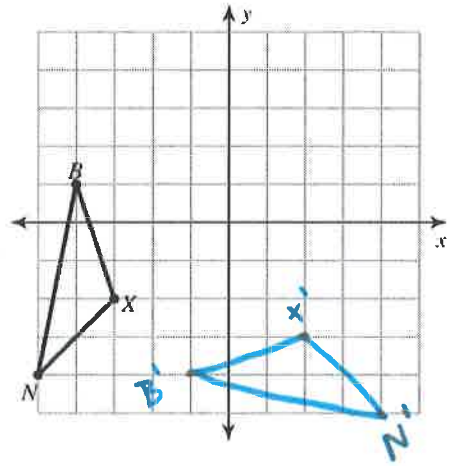
1) rotation 180° about the origin



F' (4, -2)
P' (-1, -4)
K' (-3, 0)
N' (1, 2)

3) rotation 90° counterclockwise about the origin

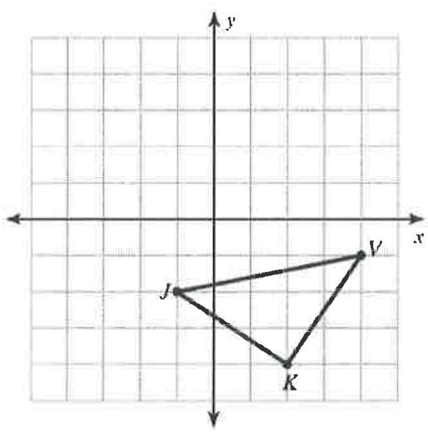
(-y, x)



B' (-1,-4)
N' (4,-5)
X' (2,-3)

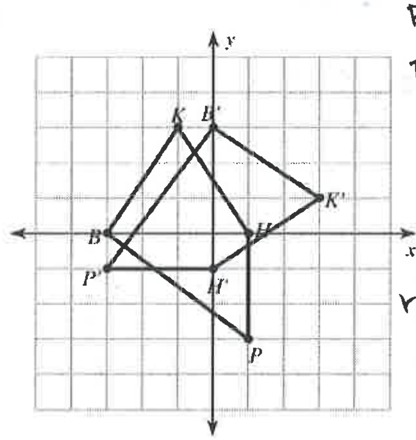
5) rotation 90° clockwise about the origin

or 270° counterclockwise?)
(y, -x)



J' (-2, 1)
K' (-4, -2)
V' (-1, -4)

Write a rule to describe each transformation.



B (-3,0)
B'(0, 3)

270° counterclockwise

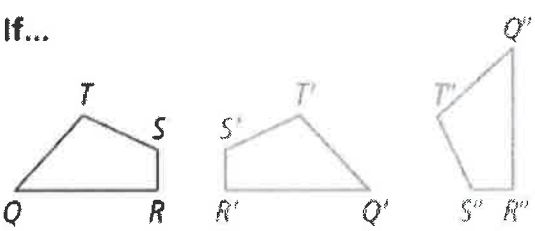
$r_{(270°, 0)}$

3.4 Classification of Rigid Motion

Glide Reflection: a transformation consisting of a translation combined with a reflection about a line parallel to the direction of the translation.

The composition of two or more rigid motions is a rigid motion.

If...



M: *QRST* → *Q′R′S′T′* and
N: *Q′R′S′T′* → *Q″R″S″T″* are rigid motions.

Then...

(*N* ∘ *M*): *QRST* → *Q″R″S″T″* is a rigid motion.

PROOF: SEE EXAMPLE 1.

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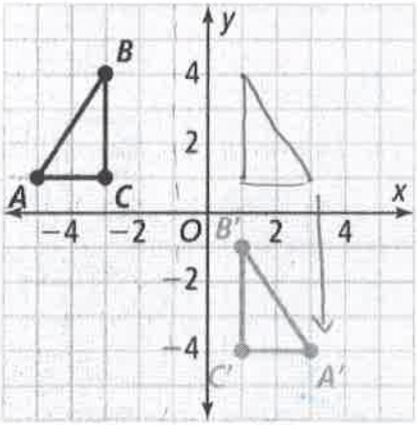
Example 1:



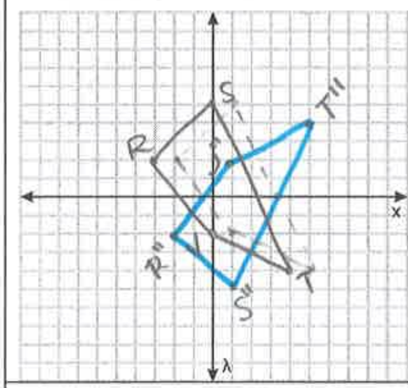
a) Is there rigid motion? Yes, shape/size the same

b) Describe the transformations.
Reflected over $x = -1$, then translated down 5

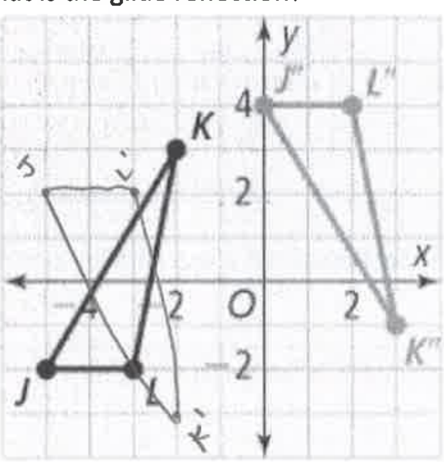
c) Write a rule for the transformations.

$$R_{x=-1} \cdot T_{\langle 0,-5\rangle}$$

Example 2: Quadrilateral RSTV has vertices R(-3, 2), S(0, 5), T(4, -4) and V(0, -2). Use the rule $T_{\langle 1,0\rangle} \cdot R_{x-axis}$.



Example 3:
What is the glide reflection?



$$R_{x\text{-axis}} \cdot T_{\langle 5,2\rangle}$$

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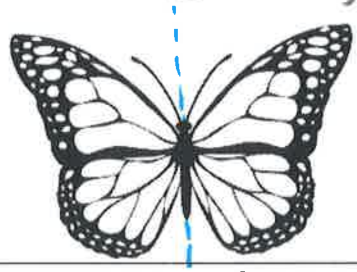
3.5 Symmetry

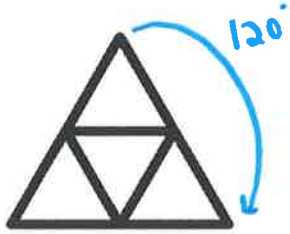
Reflectional Symmetry: A type of symmetry that maps the figure onto itself. The line of reflection is called the line of symmetry.

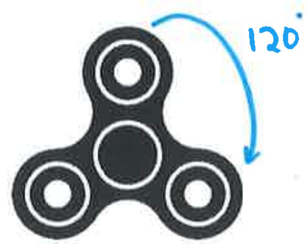
Examples:



Rotational Symmetry: A type of symmetry that maps an image onto its preimage after a rotation of less than 360°.

Examples:





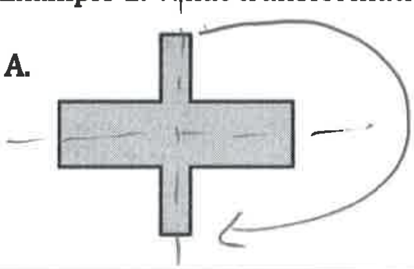
Point Symmetry: A type of symmetry where an object has rotational symmetry of 180°.

Can you think of capitalized block letters from the alphabet that have point symmetry?

S, N

Example 1: What transformation(s) will map the image onto itself?

A.

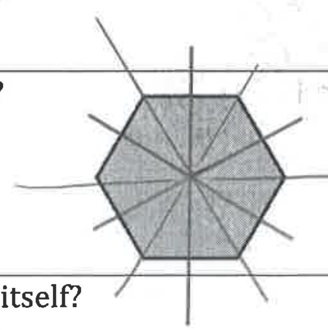


Reflectional or Rotational Symmetry

B.

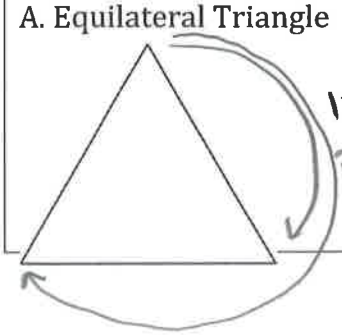
Reflectional Symmetry

Example 2: How many lines of symmetry does a regular hexagon have?



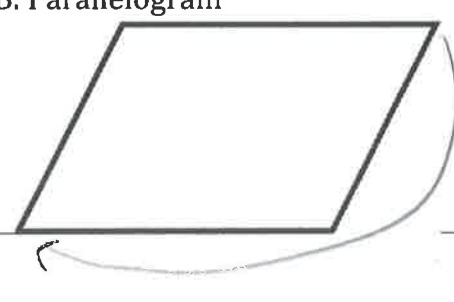
Example 3: For what angles of rotation will the figure be mapped onto itself?

A. Equilateral Triangle



120°
240°

B. Parallelogram



180°

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