KEY

6.4 Notes: Logarithmic Functions **Algebra 2**

Recall: Inverse Relations

* (x, y) becomes (y, x)
* Reflection across $y = x$
* Domain of one becomes range of other

The inverse of an exponential function is a logarithmic function

Ex. 1: Let $f(x) = 2^x$.

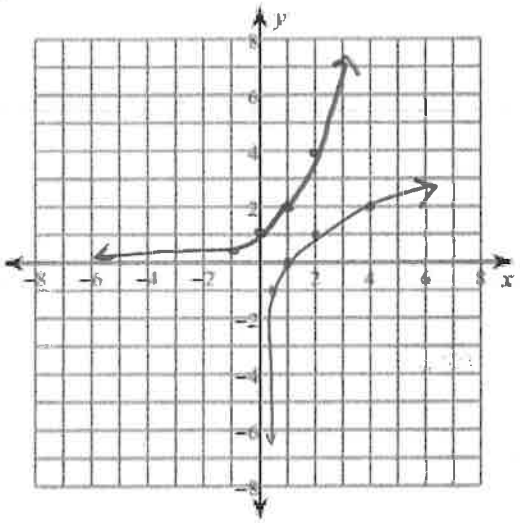
a) Graph $f(x)$.

b) Find the inverse of $f(x)$.

$x = 2^y$

$\log_2 x = y$

$f^{-1}(x) = \log_2 x$



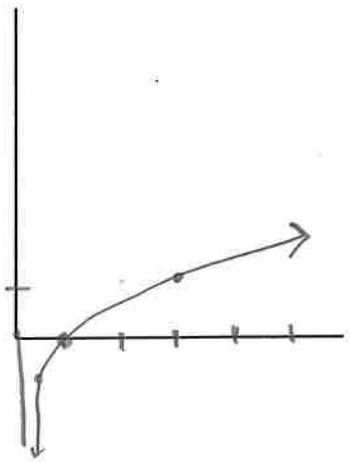
c) Use properties of inverses to graph $f^{-1}(x)$.

d) Find the key features below of $f^{-1}(x)$.

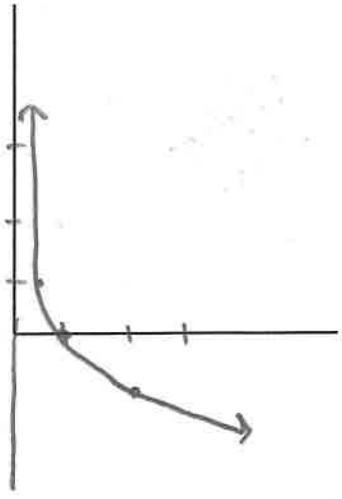
Domain: $x \in (0, \infty)$	x-int: $(1, 0)$	Asymptote: $x = 0$
Range: $y \in \mathbb{R}$	y-int: none	End Behavior: as $x \to \infty$, $y \to \infty$; as $x \to 0$, $y \to -\infty$

Parent Logarithmic Function:

$y = 3^x$ $y = \log_3 x$ $y = \left(\frac{1}{2}\right)^x$ $y = \log_{1/2} x$



x	y
$\frac{1}{3}$	-1
1	0
3	1



x	y
2	-1
1	0
$\frac{1}{2}$	1

Transformation Form for Logarithmic Functions:

vertical asymptote ↓

$$y = a \log_b (x-h) + k$$

a: vertical dilation

h: horizontal translation

k: vertical translation

b: base

if -a, reflected over xaxis / if $-(x-h)$, reflected over yaxis

We can use transformations or a table of values to graph! You are encouraged to practice both methods.

Ex. 2: Graph $h(x) = \log_2(x+3)$.

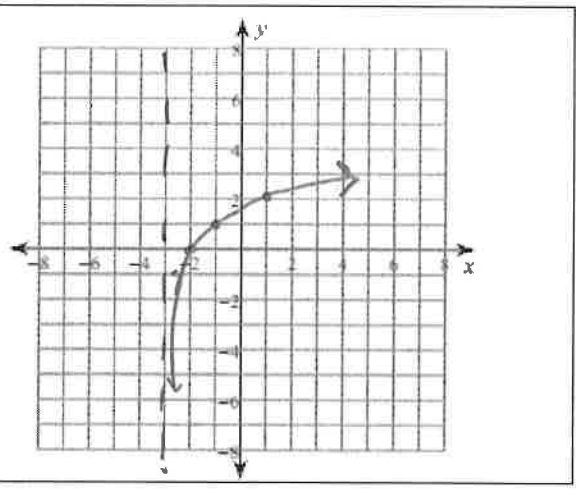
$y = \log_2 x$

x	1/2	1	2	4
~~$h(x)$~~ y	-1	0	1	2

Describe the Transformations:

left 3

Asymptote: $x = -3$



Ex. 3: Graph $k(x) = \log_7(x+5)$, find $k^{-1}(x)$, and graph $k^{-1}(x)$.

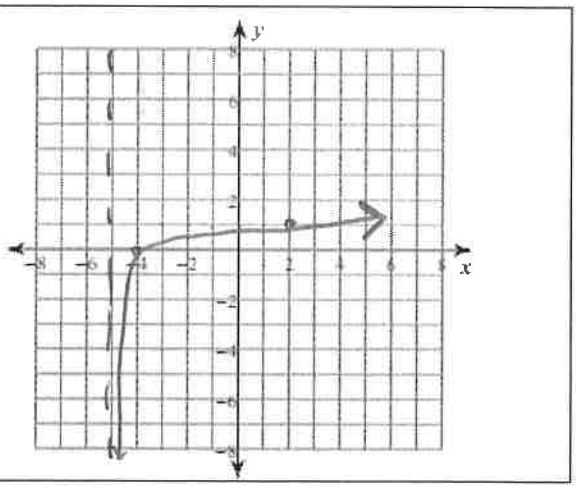
$y = \log_7 x$

x	1/7	1	7	49
~~$k(x)$~~ y	-1	0	1	2

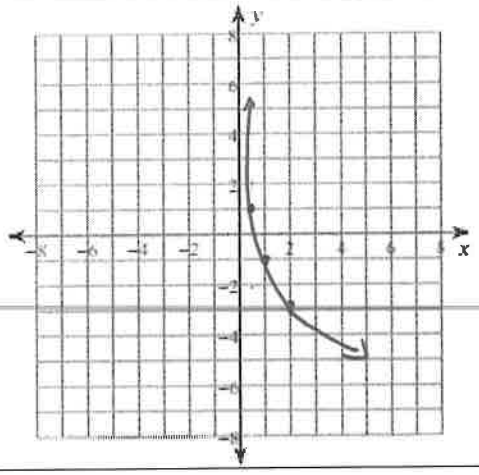
Find $k^{-1}(x)$:

$x = \log_7(y+5)$

$7^x - 5 = k^{-1}(x)$



Ex. 4: Graph $m(x) = -2\log_2 x - 1$ using transformations first, then verify with a table of values.



$\log_2 x$

x	$1/2$	1	2
~~$m(x)$~~	$-1(2) = -2$	$0(2) = 0$	$1(2) = 2$

Describe the Transformations:

- Reflected over the x axis
- Vert. Dilation of 2
- translated down 1

Ex. 5: A company relates sales revenue, R, and advertising costs, a, in dollars by: $R = 12\log(a+1) + 25$.

a) Find the average rate of revenue growth from an advertising cost of \$0.00 to \$99.00.

$R(99) = 12\log(99+1) + 25$

$= 12\log 100 + 25$ (99, 49)

$= 12(2) + 25 = 49$

$R(0) = 12\log(0+1) + 25$ (0, 25)

$= 12\log(1) + 25$

$= 12(0) + 25 = 25$

$\frac{49-25}{99-0} = \frac{24}{99} = \frac{8}{33}$

The company earns \$8 in revenue for every \$33 in adv cost

b) Explain what the inverse equation, R^{-1}, solves for, in the context of the problem.

Adv cost based on revenue

c) Write the equation for R^{-1}. $a = 12\log(R+1) + 25$

$\frac{a-25}{12} = \log(R+1)$

$10^{\frac{a-25}{12}} - 1 = R^{-1}$