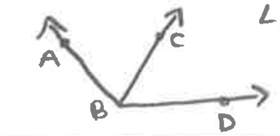
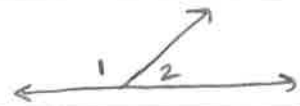
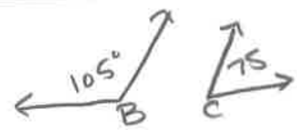
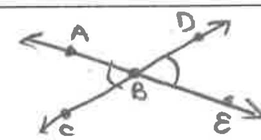
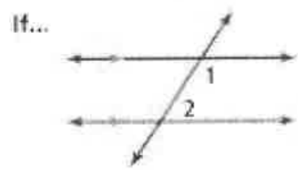
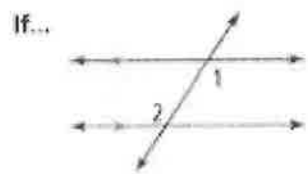
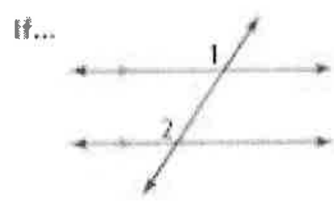
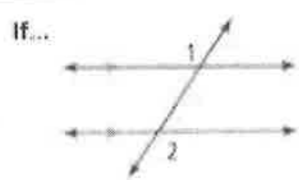
2.1-2.4 Guided Notes

2.1 Parallel and Perpendicular Lines

Description	Example
Adjacent Angles: two angles that share a common ___vertex___ and a common ___side___ and don't overlap.	 ∠ABC & ∠CBD are adjacent angles
Linear Pair of Angles: adjacent angles formed by ___two lines intersecting___	 ∠1 & ∠2 are a linear pair
Supplementary Angles: two angles whose sum is ___180°___.	 ∠B & ∠C are supplementary
Vertical Angles: opposite angles formed by two intersecting lines that share a ___vertex___, but have no common ___sides___.	 ∠ABC & ∠DBE are vertical angles

*patty paper investigation

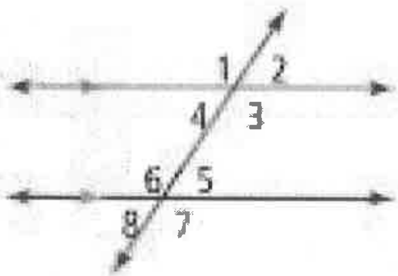
Theorems

Same Side Interior Angles Postulate (2-1): If a transversal intersects two parallel lines, then same-side interior angles are supplementary.	If... 	**Conclusion:** $m\angle 1 + m\angle 2 = 180°$
Alternate Interior Angles Thm (2-1): If a transversal intersects two parallel lines, then alternate interior angles are congruent.	If... 	**Conclusion:** $\angle 1 \cong \angle 2$
Corresponding Angles Thm (2-2): If a transversal intersects two parallel lines, then corresponding angles are congruent.	If... 	**Conclusion:** $\angle 1 \cong \angle 2$
Alternate Exterior Angles Thm (2-3): If a transversal intersects two parallel lines, then the alternate exterior angles are congruent.	If... 	**Conclusion:** $\angle 1 \cong \angle 2$

Example 1: Identify the pairs of angles of each angle type of angle from the figure below.

2.1-2.4 Guided Notes

~~Corresponding Angles:~~



~~Alternate Interior Angle:~~
~~Alternate Exterior Angles:~~

Corresponding Angles

∠2 & ∠5
∠3 & ∠7
∠8 & ∠4
∠6 & ∠1

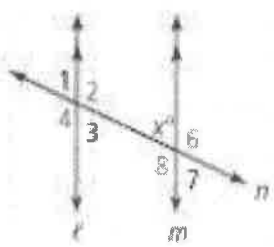
Alternate Interior Angles

∠4 & ∠5
∠3 & ∠6

Alternate Exterior Angles

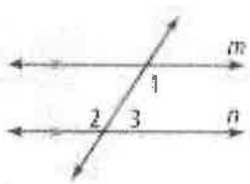
∠8 & ∠2
∠1 & ∠7

Example 2: How can you express each of the numbered angles in terms of x?



∠1 = x
∠2 = 180 − x
∠3 = x
∠4 = 180 − x
∠6 = 180 − x
∠7 = x
∠8 = 180 − x

Example 3: Prove the Alternate Interior Angles Theorem.



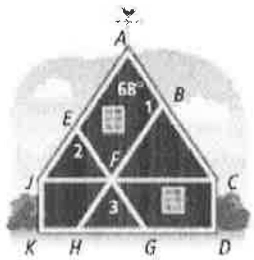
Given: $m \parallel n$

Prove: $\angle 1 \cong \angle 2$

Statement	Reason
m ∥ n	given
∠1 & ∠3 are supplementary	Same Side Interior Angles Post
m∠1 + m∠3 = 180°	Definition of Supplementary ∠s
m∠2 + m∠3 = 180°	Angle Addition Postulate
m∠1 + m∠3 = m∠2 + m∠3	Transitive Property
m∠1 = m∠2	Subtraction Property of Eq.
∠1 ≅ ∠2	Definition of Congruence

2.1-2.4 Guided Notes

Example 4: The white trim shown for the wall of a barn should be constructed so that $\overline{AC} \parallel \overline{EG},\ \overline{JA} \parallel \overline{HB}\ and\ \overline{JC} \parallel \overline{KG}$ Find the measure of the missing angles.



$m\angle 1 =$ 112° ← 180 - 68

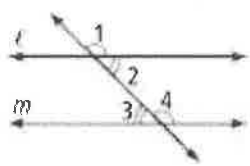
$m\angle 2 =$ 68°

$m\angle 3 =$ 68°

2.2 Proving Lines Parallel

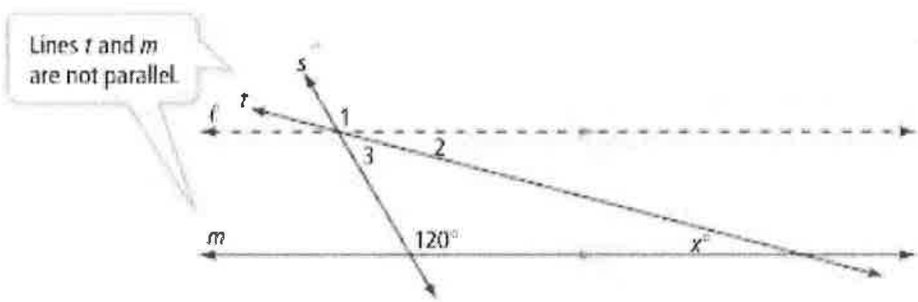
Warm-Up

Analyze the diagram to see if line *l* is parallel to line *m*. Is there enough information to determine if the lines are parallel?



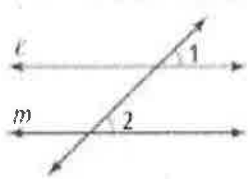
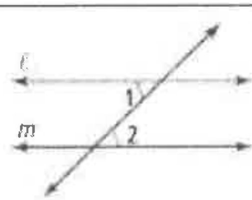
Yes.
Converse of Corresponding Angles &
Alternate Interior Angles

Example 1: Line *l* is parallel to line *m*, find the missing angle measures. Explain your reasoning.

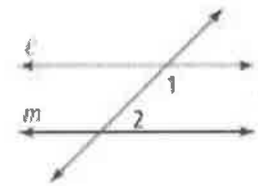
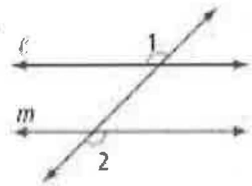
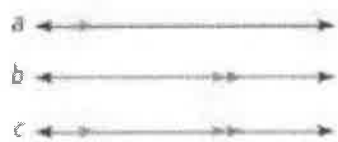
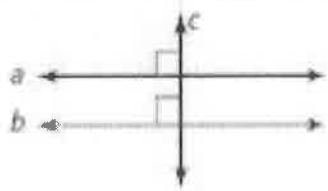


m∠1 = 120°
m∠2 = x

Theorems

Converse of the Corresponding Angles Thm (2-4): If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.	If... 	**Conclusion:** l ∥ m
Converse of the Alternate Interior Angles Thm (2-5): If two lines and a transversal form alternate interior angles that are congruent, then the lines are parallel.	If... 	**Conclusion:** l ∥ m

2.1-2.4 Guided Notes

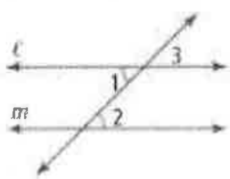
Converse of the Same-Side Interior Angles Postulate(2-6): If two lines and a transversal form same-side interior angles that are supplementary, then the lines are parallel.	If... $m\angle 1 + m\angle 2 = 180$ 	**Conclusion:** $\ell \parallel m$
Converse of the Alternate Exterior Angles Thm (2-7): If two lines and a transversal form alternate exterior angles that are congruent, then the lines are parallel.	If... 	**Conclusion:** $\ell \parallel m$
Thm (2-8): If two lines are parallel to the same line, then they are all parallel to each other.	If... 	**Conclusion:** $a \parallel b$ $a \parallel c$
Thm (2-9): If two lines are perpendicular to the same line, then they are parallel to each other.	If... 	**Conclusion:** $a \parallel b$

Flow Chart Proof

Write a flow chart proof to prove the Converse of the Alternate Interior Angles Theorem.

Given: $\angle 1 \cong \angle 2$

Prove: $\ell \parallel m$



oof:

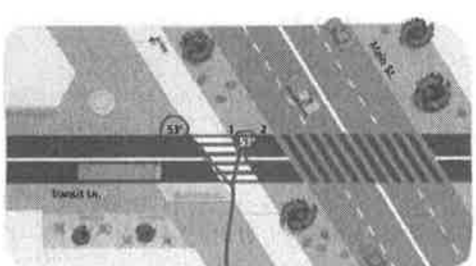
$\angle 1 \cong \angle 2$ — Given

$\angle 1 \cong \angle 3$ — Vertical Angle Theorem

$\angle 2 \cong \angle 3$ — Transitive Prop. of $\cong$

$\ell \parallel m$ — Converse of Corresponding Angle Theorem

Example 1: Determine whether the lines are parallel.
The edges of a new sidewalk must be parallel in order to meet accessibility requirements. Concrete is poured between straight strings. How does an inspector know that the edges of the sidewalk are parallel?



$m\angle 1 = 127°$

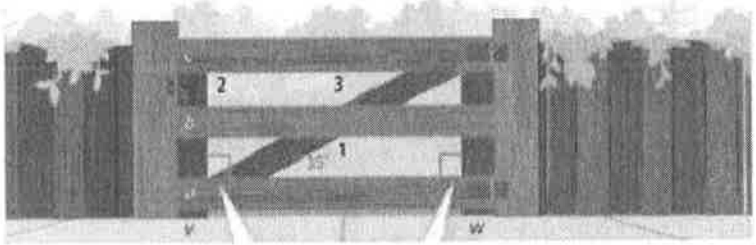
$m\angle 2 = 53°$

Since these are both 53°, Converse of Alternate Exterior Angles says they are parallel

2.1-2.4 Guided Notes

Example 2:

a) When building a gate, how does Bailey know that the vertical boards v and w are parallel?



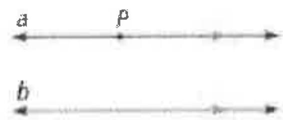
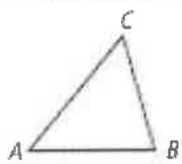
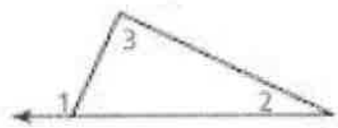
Theorem 2-9 because two lines are perpendicular to the same line

b) What should the m∠1 to ensure board b is parallel to board a?

$m\angle 1 = 145°$

2.3 Parallel Lines and Triangle Angle Sums

Theorems

Theorem (2-10): Through a point not on a line, there is one and only one line parallel to the given line.	If... 	**Conclusion:** line a is the only line parallel to line b through P.
Triangle Angle Sum Thm (2-11): The sum of the measures of all the angles of a triangle is 180°.	If... 	**Conclusion:** $m\angle A + m\angle B + m\angle C = 180°$
Triangle Exterior Angle Thm (2-12): The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.	If... 	**Conclusion:** $m\angle 1 = m\angle 3 + m\angle 2$

Example 1: Determine the values of x and y.

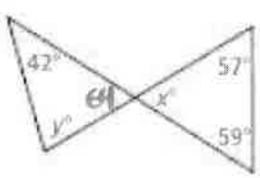
a.

84°

x = 49°

y = 19°

b.



x = 180 - 57 - 59 =

x = 64°

y = 180 - 42 - 64

y = 74°

c.

x = 180 - 23 - 54

x = 103°

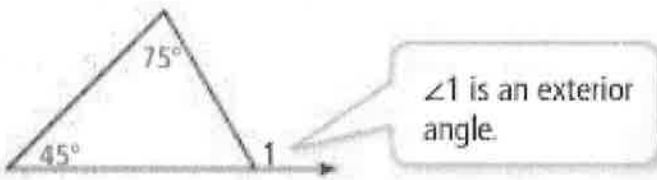
y = 77 + 36

y = 113°

2.1-2.4 Guided Notes

Example 2: What is the missing angle measure in each figure?

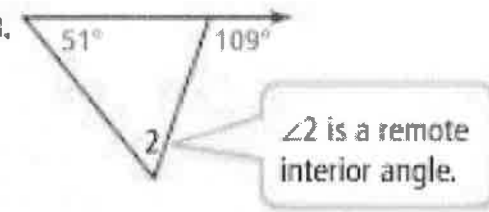
A.



$m\angle 1 = 75 + 45$

$m\angle 1 = 120°$

B.



$m\angle 2 = 109 - 51$

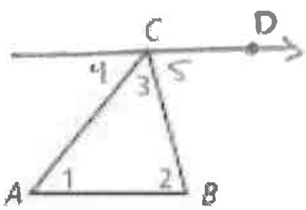
$m\angle 2 = 58°$

Example 2:

Prove the Triangle Angle-Sum Theorem.

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



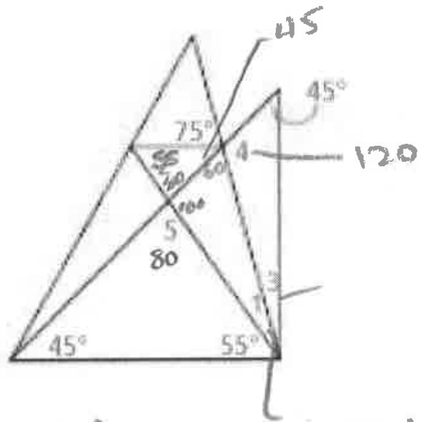
Plan: Draw a line through C, because a straight angle measures 180°. This line should be parallel to the line containing $\overline{AB}$ so that an ____.

Statement	Reason
$\triangle ABC$	given
$\overline{CD} \parallel AB$	Theorem 2-10
$m\angle 3 + m\angle 4 + m\angle 5 = 180°$	Angle Addition Postulate
$m\angle 2 = m\angle 5$, $m\angle 1 = m\angle 4$	Alternate Interior Angle Thm
$m\angle 1 + m\angle 2 + m\angle 4 = 180°$	Substitution Prop of Eq

2.1-2.4 Guided Notes

Example 4:

Cheyenne built this display for her ornament collection. Each shelf is parallel to the base. She recalls only the angle measures shown in the diagram. Now she wants to build another just like it. What are the measures of $\angle 1$, $\angle 2$, and $\angle 3$?



$m\angle 1 = 20°$

$m\angle 2 = 55°$

$m\angle 3 = 15°$

$m\angle 4 = 120°$

$m\angle 5 = 80°$

2.4 Slopes of Parallel and Perpendicular Lines

Review Vocabulary: Tell your partner what you remember about slope.

Slope: $\dfrac{\text{Rise}}{\text{Run}} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

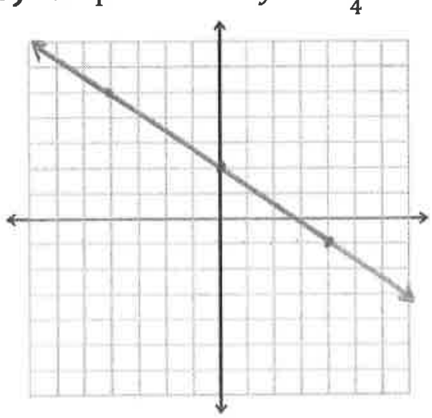
Positive Slope: increases from left to right

Negative Slope: decreases from left to right

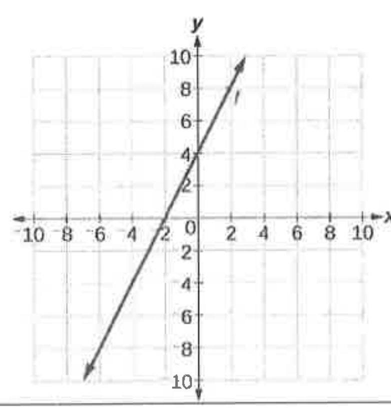
Equation of a line: $y = \underbrace{m}_{\text{slope}}x + b$ ← y-intercept

Example 1:

a) Graph the line: $y = -\frac{3}{4}x + 2$



b. Write the equation of the line.



$y = 2x + 4$

2.1-2.4 Guided Notes

Practice: Find the slope between the set of points.

a. (2, 2) and (5, 4)

$m = \frac{4-2}{5-2} = \frac{2}{3}$

b. (-1, -2) and (2, 1)

$m = \frac{1-(-2)}{2-(-1)} = \frac{3}{3} = 1$

Discuss: Nadia and Jake begin climbing to the top of a 100-foot monument along two different sets of stairs at the same rate. The table show their distances above ground level after a number of steps.

Nadia

Steps	Distance (ft)
1	2
3	3
17	10
25	14

2 1

Jake

Steps	Distance (ft)
1	5
7	8
15	12
29	19

6 3

a) How many feet does each student climb after 10 steps? Explain.

Nadia: 0.5 ft/step → 5 ft

Jake: 0.5 ft/step → 5 ft

b) Will Nadia and Jake be at the same height at the same number of steps? Explain. No Jake started higher

c) What would you expect the graphs of each to look like given your answers to parts A and B? Explain.

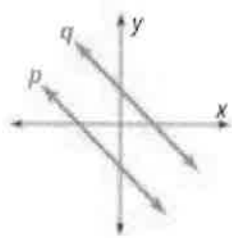
increasing at the same rate with Jake starting higher

Theorems

Theorem (2-13): Two non-vertical lines are parallel if and only if their slopes are parallel.

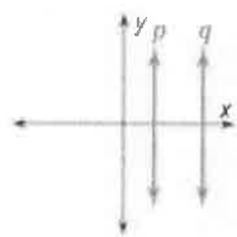
Any two vertical lines are parallel.

If... p and q are both not vertical



Then p ∥ q if p & q have the same slope

If... p and q are both vertical

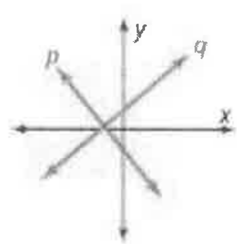


Then p ∥ q

Theorem (2-14): Two non-vertical lines are perpendicular if and only if the product of their slopes is -1.

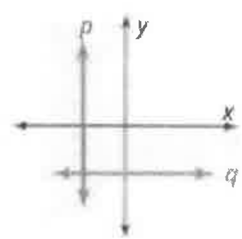
A vertical line and a horizontal line are perpendicular to each other.

If... p and q are both not vertical



$p \perp q$ if & only if the product of the slopes is -1

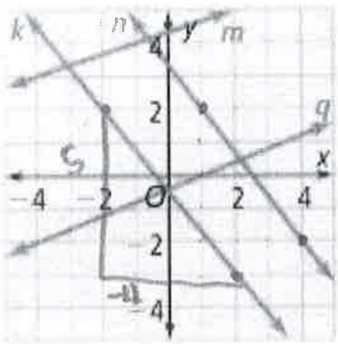
If... one of p and q is vertical and the other is horizontal



$p \perp q$

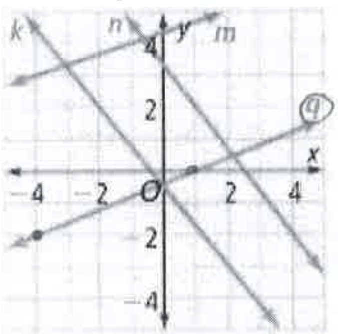
2.1-2.4 Guided Notes

Example 1: Check Parallelism.
Are lines k and n parallel? Justify your answer.



k has a slope of $-\frac{5}{4}$
n has a slope of $-\frac{4}{3}$
so no, they dont have the same slope.

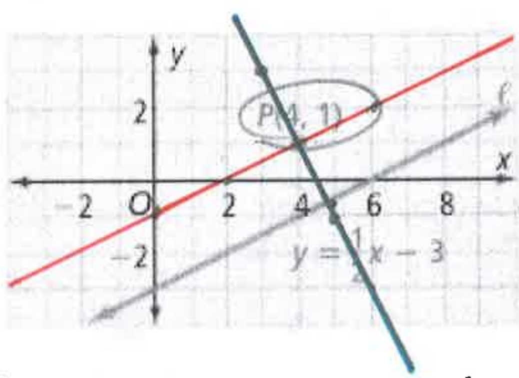
Example 2: Check Perpendicularity.
Are lines q and k perpendicular? Justify your answer.



k's slope is $-\frac{5}{4}$
q's slope is $\frac{2}{5}$
No, they dont have opposite reciprocal slopes

Example 3: Write the equation of parallel and perpendicular lines.

a) What is an equation of the line through P that is parallel to line *l*?



Step 1: Find the slope of the given line.

$\frac{1}{2}$

Step 2: Solve for the y-intercept by substituting the slope and point into the equation y = mx +b.

$y = \frac{1}{2}x + b$ $b = -1$
$1 = \frac{1}{2}(4) + b$
$1 = 2 + b$

$y = \frac{1}{2}x - 1$

Step 3: Write the equation of the line.

b) What is the equation of the line through P perpendicular to line *l*? -2

2.1-2.4 Guided Notes

Step 1: Identify the slope of the perpendicular line.

-2

Step 2: Solve for the y-intercept by substituting the slope and point into the equation y = mx + b

$1 = -2(4) + b$

$1 = -8 + b$

$9 = b$

$y = -2x + 9$

Step 3: Write the equation of the line.

$y = -2x + 9$

You try!

What are the equations of the lines parallel and perpendicular to the given line k through point T?

a. $y = -3x + 2;\ T(3,1)$

parallel:

$y = -3x + b$

$1 = -3(3) + b$

$1 = -9 + b$

$10 = b$

$y = -3x + 10$

Perpendicular:

$y = \frac{1}{3}x + b$

$1 = \frac{1}{3}(3) + b$

$1 = 1 + b$

$0 = b$

$y = \frac{1}{3}x$

b. $y = \frac{3}{4}x - 5;\ T(12,-2)$

parallel:

$y = \frac{3}{4}x + b$

$-2 = \frac{3}{4}(12) + b$

$-2 = 9 + b$

$-11 = b$

$y = \frac{3}{4}x - 11$

perpendicular

$y = -\frac{4}{3}x + b$

$-2 = \frac{-4}{3}(12) + b$

$-2 = -16 + b$

$14 = b$

$y = -\frac{4}{3}x + 14$