5.6 Notes - Inverse Relations/Functions

Inverse Relation formed when the roles of the independent & dependent variables are reversed.

Relation

x	y
-2	1
-1	4
0	3

→

Inverse Relation

x	y
1	-2
4	-1
3	0

switch x & y values

Inverse Function: inverse relation in which it is a function (every x-value has one y-value).

x	y
-2	1
-1	4
1	1

→

x	y
1	-2
4	-1
1	1

2 y-values for one x-value

To find inverse, switch x & y.

x	y
-2	4
3	6
1	2

→

x	y
4	-2
6	3
2	1

Find the inverse of $f(x) = 2x + 1$

$x = 2y + 1$ ① Switch x & y

$x - 1 = 2y$ ② Solve for y

$\frac{1}{2}x - \frac{1}{2} = y \rightarrow f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$

Inverse function notation

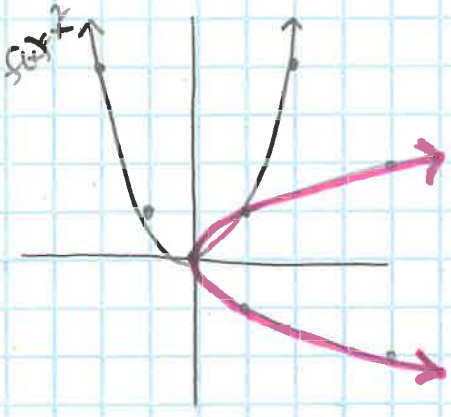
② Find the inverse.

$f(x) = x^2$

$\sqrt{x} = \sqrt{y^2}$

$\pm\sqrt{x} = y$ ← Is this a function

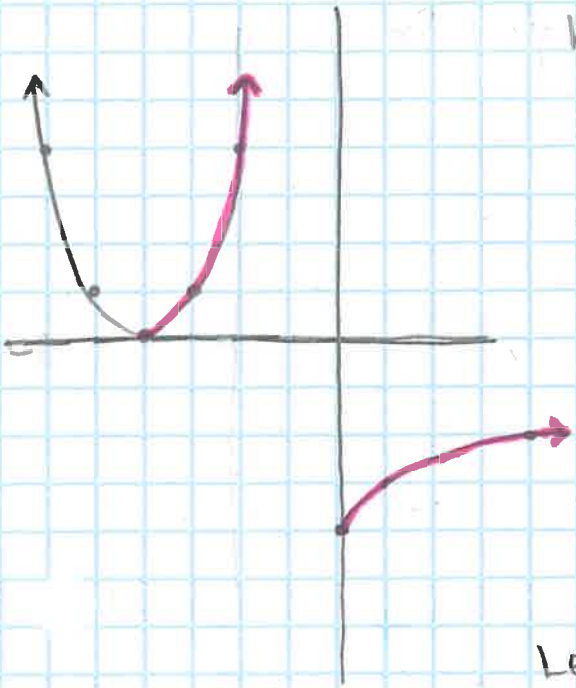
*Since its not a function leave as $y=$



It would be a function when $x \in [0, \infty)$

so $f^{-1}(x) = \sqrt{x} \quad x \in [0, \infty)$

③ Find the inverse & find the domain for $k(x)$ that makes it a function



$k(x) = x^2 + 8x + 16$

$k(x) = (x+4)(x+4)$ Factor

$k(x) = (x+4)^2$ Switch x & y

$x = (y+4)^2$

$\pm\sqrt{x} = y+4$

$\pm\sqrt{x} - 4 = y$

Let $k(x)$ have domain $x \in [-4, \infty)$
then the inverse is $y = \sqrt{x} - 4$

* For any quadratic restrict the domain about the vertex.

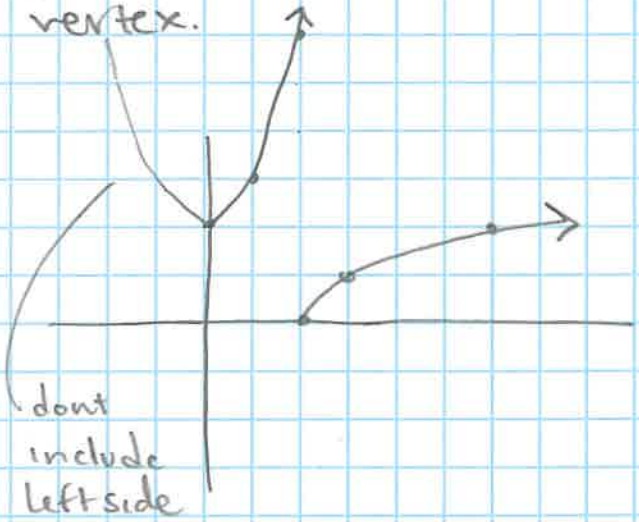
$f(x) = \sqrt{x-2}$

$x = \sqrt{y-2}$

$x^2 = y - 2$

$x^2 + 2 = y$

$f^{-1}(x) = x^2 + 2$ when $x \in [0, \infty)$



$f(x) = 2 - \sqrt[3]{x+1}$

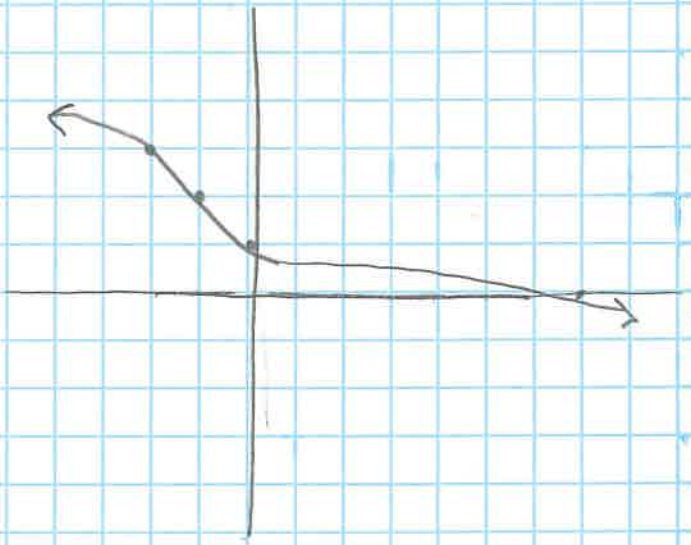
$x = 2 - \sqrt[3]{y+1}$

$x - 2 = -\sqrt[3]{y+1}$

$-x + 2 = \sqrt[3]{y+1}$

$(-x+2)^3 = y + 1$

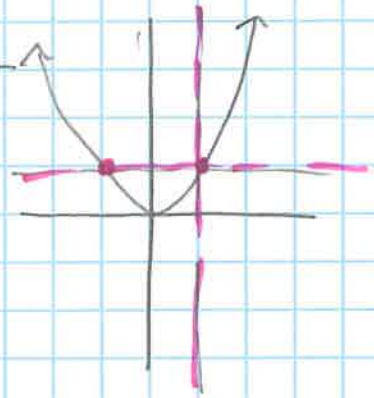
$(-x+2)^3 - 1 = f^{-1}(x)$



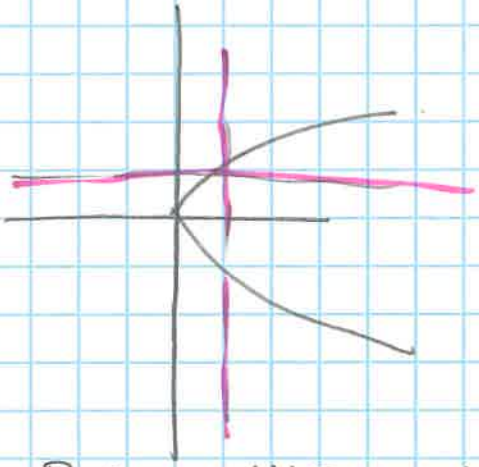
Recall vertical line test determines if a relation is a function.

→ Inverse reverse x & y so horizontal line test determines if inverse will be a function

Ex



Passes VLT but not HLT



Passes HLT but not VLT so inverse is function but not original.

$f(f^{-1}(x)) = x$ or $f^{-1}(f(x)) = x$

proves inverses

$g(x) = \frac{1}{2}x - \frac{5}{2}$ $\quad$ $f(x) = 2x + 5$

$(f(g(x)) = 2\left(\frac{1}{2}x - \frac{5}{2}\right) + 5$

$= x - 5 + 5$

$f(g(x)) = x$ so $f(x)$ & $g(x)$ are inverses

Determine if the functions are inverses

$m(x) = \sqrt[3]{x-1}$ $\quad n(x) = x^3 + 1$

$m(n(x)) = \sqrt[3]{x^3+1-1}$

$= \sqrt[3]{x^3}$

$m(n(x)) = x$ yes inverses

A sculpture artist is making an ice sculpture of Earth. He created a mold that can hold 4.5 L of ice. What will the radius of the ice sculpture be?

$V_{sphere} = \frac{4}{3}\pi r^3$

$\frac{3}{4}V = \pi r^3$ Solve to find r.

$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3}$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

$1\text{ L} = 1000\text{ cm}^3$

$4.5(1000) = 4500\text{ cm}^3$

$r = \sqrt[3]{\frac{3(4500)}{4\pi}}$

$r \approx 10.2\text{ cm}$