

Recall: Exponential Growth vs. Exponential Decay

Exponential Growth	Exponential Decay
Equation: $A(t) = P(1+r)^t$	Equation: $A(t) = P(1-r)^t$
Growth Rate: $r > 0$	Decay Rate: $0 < r < 1$
Synonyms in Application Problems: Appreciation, Increases	Synonyms in Application Problems: Depreciate, Decreases

Ex. 1: Modeling Population Growth. In 2015, the population of a small town was 8,000. The population is expected increase at a rate of 2.5% each year. Write a function $A(t)$ to represent the population, A , after t years, then use your equation to estimate the population in 2018.

$$A(t) = 8000(1.025)^t \quad 2018 \rightarrow t=3$$

$$A(3) = 8000(1.025)^3 = \$8615.125$$

Compound interest: when interest is earned on interest that has already been earned. (Example: if you earn monthly interest, that interest becomes part of the new principal for the second month)

Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

P : initial amount

r : interest rate

n : number of times compounded

t : time

Ex. 2A: Using Compound Interest. Tamira has \$5,000 to invest and is looking at which account will give her the best return after 3 years. All accounts earn 4%. Help her choose.

Compounding Period(s)	Compounded Annually	Compounded Semi-annually
Equation	$A = 5000\left(1 + \frac{.04}{1}\right)^{1 \cdot t}$	$A = 5000\left(1 + \frac{.04}{2}\right)^{2 \cdot t}$
Value after 3 years	$A = 5000\left(1 + \frac{.04}{1}\right)^3 = \5624.32	$A = 5000\left(1 + \frac{.04}{2}\right)^6 = \5636.81
	Compounded Quarterly	Compounded Monthly
Equation	$A = 5000\left(1 + \frac{.04}{4}\right)^{4 \cdot t}$	$A = 5000\left(1 + \frac{.04}{12}\right)^{12 \cdot t}$
Value after 3 years	$A = 5000\left(1 + \frac{.04}{4}\right)^{12} = \5634.13	$A = 5000\left(1 + \frac{.04}{12}\right)^{36} = \5636.36

Choice: compounded monthly

What pattern do you notice? more times compounded, higher balance.

Ex. 2B: Analyzing Compound Interest. Tamira found two new options with higher rates. In option 1, Tamira can invest \$5,000 into an account that earns 5.5% compounded annually. In option 2, Tamira can invest \$5,000 into an account that earns 5% compounded monthly. Help her choose between these options and justify your reasoning.

Opt 1

$$A = 5000(1 + .055)^t$$

Opt 2

$$A = 5000\left(1 + \frac{.05}{12}\right)^{12t}$$

GRAPH on grapher to compare

Ex. 3: Creating an Exponential Model with Two Points. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth \$31,000 in 1954 and \$35,000 in 1955. Use the data to determine an exponential model that describes the value of the land. Next, use your model to estimate the value of the land in 1970.

① Identify 2 points.

$$(4, 31000) \quad (5, 35000)$$

② Plug into equations $y = a \cdot b^x$

$$\begin{array}{r} 35000 = a \cdot b^5 \\ 31000 = a \cdot b^4 \end{array}$$

$$1.129 = b$$

$$\begin{array}{r} 35000 = a \cdot 1.129^5 \\ 35000 = a(1.834) \end{array}$$

$$19080.88 = a$$

③ Divide functions to find b.

④ Plug b into one equation to find a.

$$y(20) = \$216,011.68$$

$$y = 19080.88(1.129)^x$$

You Try! On January 1st, John listed his old Nintendo console and more than 100 original games on Craig's List. He's been receiving emails at an exponential rate, but has been too busy to check. On January 6th, John received 1,000 emails. On January 11th, John received 1,400 emails. Write an exponential function to model the number of emails John receives on each day since January 1st.

$$(5, 1000) \quad (10, 1400)$$

$$\begin{array}{r} 1400 = a \cdot b^{10} \\ 1000 = a \cdot b^5 \end{array}$$

$$(1.4 = b^5)^{1/5}$$

$$1.0696 = b$$

$$1400 = a(1.0696)^{10}$$

$$1400 = a(1.96)$$

$$714.286 = a$$

$$y = 714.286(1.0696)^x$$