

9.5 Notes

Completing the Square - when a doesn't equal 1

What do we need to add to these expressions to complete the square (create a perfect square)?

1. $2x^2 + 16x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}(x + \underline{\hspace{1cm}})^2$

2. $3x^2 - 6x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}(x - \underline{\hspace{1cm}})^2$

3. $2x^2 + 10x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}(x + \underline{\hspace{1cm}})^2$

4. $2x^2 - 20x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}(x - \underline{\hspace{1cm}})^2$

How did you find the c value in $ax^2 + bx + c$ when there is a number in front of a?

Ex. 1: Solve by completing the square.

$$2x^2 - 12x + 16 = 0$$

- 1) Isolate the $ax^2 + bx$ on one side.
- 2) Factor out the \underline{a} from $ax^2 + bx$
- 3) Determine what you need to add in the parenthesis to the right side to complete the square. **To keep the equation balanced, you need to add the same amount to each side, but be careful!!!**
- 4) Write the left side as a squared binomial.
- 5) Solve the quadratic by taking the square root of both sides and continue to solve.

Ex. 2: Solve by completing the square.

$$4x^2 + 16x - 8 = 0$$

You Try! Solve by completing the square.

$$5x^2 + 10x - 15 = 0$$

Ex. 3: Write in Vertex Form by Completing the Square.

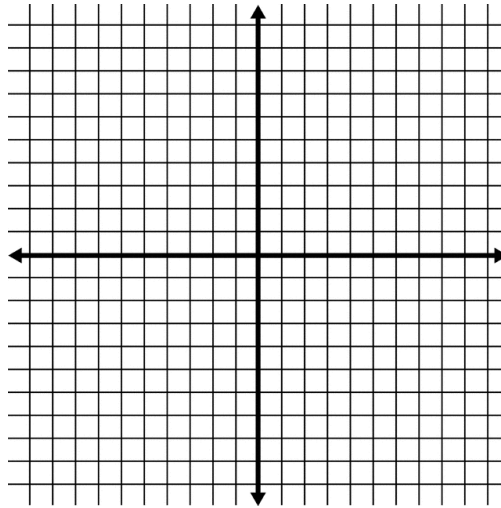
Vertex Form: _____

$$y = 2x^2 - 8x + 11$$

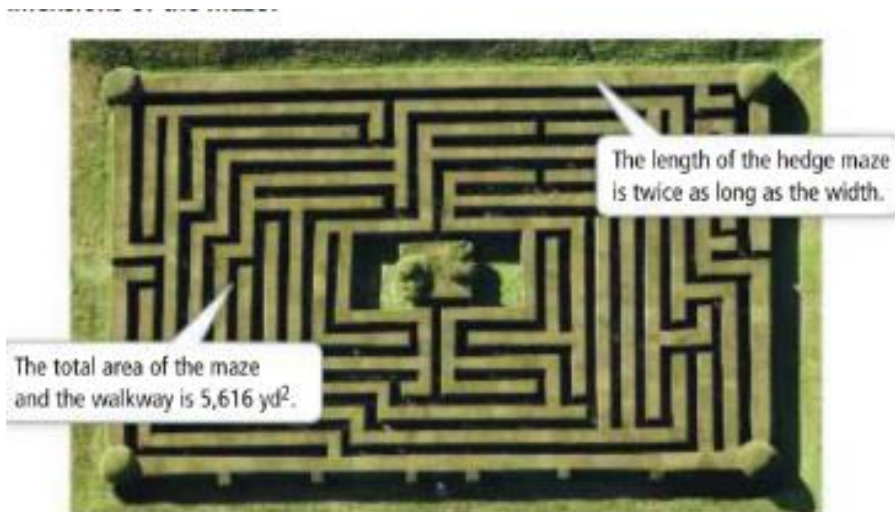
- 1) Isolate the $(ax^2 + bx)$
- 2) Factor out the a from $ax^2 + bx$
- 3) Complete the square in the parenthesis. **Subtract outside the parenthesis to keep the equation balanced. Again, remember to multiply what you are subtracting by a .**
- 4) Simplify the parenthesis to a squared binomial

Ex. 4: Write in vertex form and graph.

$$y = 2x^2 - 4x + 5$$



Ex.5: A heddge maze has a 2 yard wide walkway around it. What are the dimensions of the maze with the walkway, if the length of the hedge maze is twice as long as the width?



$$x = -56 \quad x = 50$$

What do we need to add to these expressions to complete the square (create a perfect square)?

$$1. \quad 2x^2 + 16x + 32 = 2(x + 4)^2$$

$$2(x^2 + 8x + 16) = 2(x + 4)^2$$

$$\frac{8}{2} = 4 \quad 4^2 = 16$$

$$2x^2 + 16x + \boxed{32}$$

$$2. \quad 3x^2 - 6x + 3 = 3(x - 1)^2$$

$$3(x^2 - 2x + 1) = 3(x - 1)^2$$

$$-\frac{2}{2} = -1 \quad (-1)^2 = 1$$

$$3x^2 - 6x + \underline{3}$$

How did you find the c value in $ax^2 + bx + c$ when there is a number in front of a?

- 1) factored out a
- 2) Divide b by 2 & square it
- 3) Distribute a back through

Ex. 1: Solve by completing the square.

$$2x^2 - 12x + 16 = 0$$

$$-16 \quad -16$$

$$2x^2 - 12x = -16$$

$$\underline{2}(x^2 - 6x + \underline{9}) = -16 + \underline{2(9)}$$

$$-\frac{6}{2} = -3 \quad (-3)^2 = 9$$

$$2(x - 3)^2 = -16 + 18$$

$$\frac{2(x - 3)^2}{2} = \frac{2}{2}$$

$$\sqrt{(x - 3)^2} = \sqrt{1}$$

$$x - 3 = \pm 1$$

$$+3 \quad +3$$

$$\boxed{\begin{array}{l} x = 4 \\ x = 2 \end{array}}$$

1) Isolate the $ax^2 + bx$ on one side.

2) Factor out the a from $ax^2 + bx$

3) Determine what you need to add in the parenthesis to the ~~right~~ ^{left} side to complete the square. **To keep the equation balanced, you need to add the same amount to each side, but be careful!!!**

4) Write the left side as a squared binomial.

5) Solve the quadratic by taking the square root of both sides and continue to solve.

Ex. 2: Solve by completing the square.

$$4x^2 + 16x - 8 = 0$$

$$\begin{aligned}
 &4x^2 + 16x = 8 \\
 &4(x^2 + 4x + \underline{4}) = 8 + \underline{4(4)} \\
 &\quad \quad \quad \frac{4}{2} = 2 \quad (2)^2 = 4 \\
 &4(x + 2)^2 = 8 + 16 \\
 &\quad \quad \quad \frac{4(x+2)^2}{4} = \frac{24}{4} \\
 &\quad \quad \quad \sqrt{(x+2)^2} = \sqrt{6} \\
 &\quad \quad \quad x+2 = \pm \sqrt{6} \\
 &\quad \quad \quad \quad \quad -2 \quad \quad -2 \\
 &\quad \quad \quad \boxed{x = -2 \pm \sqrt{6}}
 \end{aligned}$$

You Try! Solve by completing the square.

$$5x^2 + 10x - 15 = 0$$

$$\begin{aligned}
 &5x^2 + 10x = 15 \\
 &5(x^2 + 2x + \underline{1}) = 15 + \underline{1(5)} \\
 &\frac{5(x+1)^2}{5} = \frac{20}{5} \\
 &\sqrt{(x+1)^2} = \sqrt{4} \\
 &\quad \quad \quad x+1 = \pm 2 \\
 &\quad \quad \quad \quad \quad -1 \quad \quad -1 \\
 &\quad \quad \quad \boxed{x = 1 \quad \text{or} \quad x = -3}
 \end{aligned}$$

Ex. 3: Write in Vertex Form by Completing the Square.

Vertex Form: $y = a(x-h)^2 + k$

$$y = 2x^2 - 8x + 11$$

$$\begin{aligned}
 &y = (2x^2 - 8x) + 11 \\
 &y = 2(x^2 - 4x + \underline{4}) + 11 - \underline{2(4)} \\
 &\quad \quad \quad \frac{4}{2} = 2 \quad 2^2 = 4
 \end{aligned}$$

$$y = 2(x+2)^2 + 11 - 8$$

$$y = 2(x+2)^2 + 3$$

- 1) Isolate the $(ax^2 + bx)$
- 2) Factor out the a from $ax^2 + bx$
- 3) Complete the square in the parenthesis. **Subtract outside the parenthesis to keep the equation balanced. Again, remember to multiply what you are subtracting by a .**
- 4) Simplify the parenthesis to a squared binomial

Ex. 4: Write in vertex form and graph.

$$y = 2x^2 - 4x + 5$$

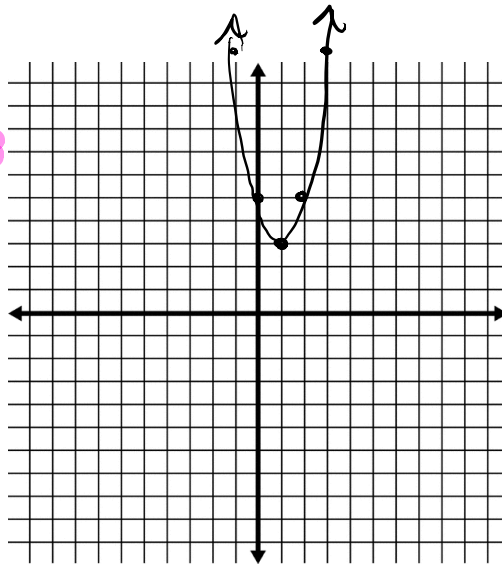
$$y = (2x^2 - 4x) + 5$$

$$y = 2(x^2 - 2x + \frac{1}{2}) + 5 - (1 \cdot 2)$$

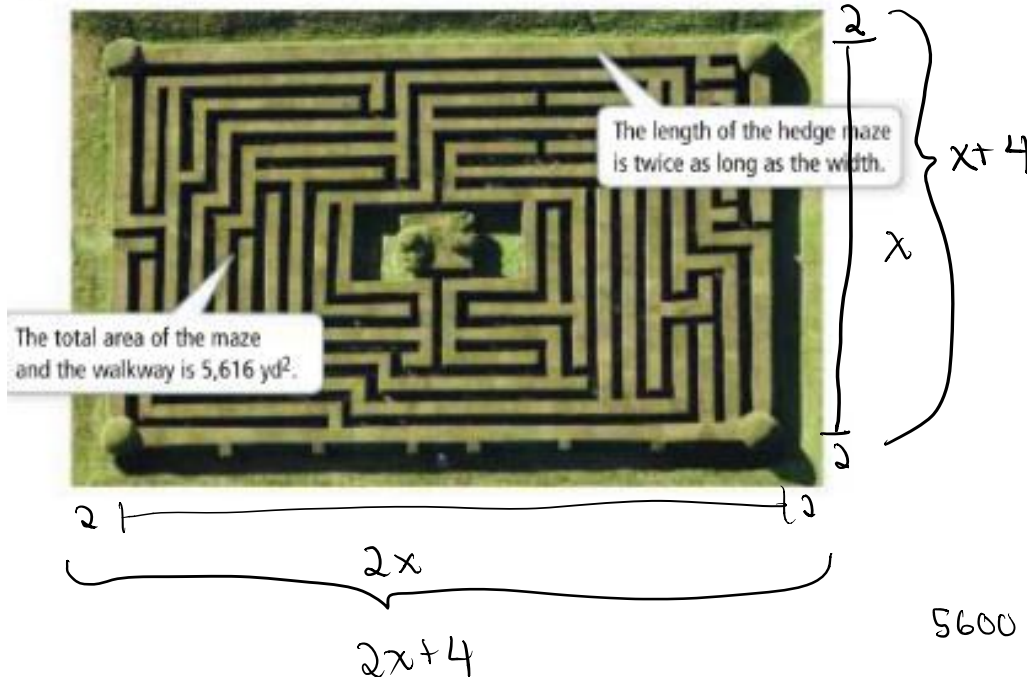
\downarrow
 $\frac{-2}{2} = -1$ $(-1)^2 = 1$

$$y = 2(x - 1)^2 + 3$$

Vertex: (1, 3)



Ex. 5: A hedge maze has a 2 yard wide walkway around it. What are the dimensions of the maze with the walkway, if the length of the hedge maze is twice as long as the width?



$$\text{Area} = 5616 \text{ yd}^2$$

$$\text{Area} = l \cdot w$$

$$5616 = (2x + 4)(x + 4)$$

$$5616 = 2x^2 + 8x + 4x + 16$$

$$5616 = \frac{2x^2 + 12x + 16}{-16}$$

$$5600 = 2x^2 + 12x$$

$$5600 + 2(9) = 2(x^2 + 6x + 9)$$

$$\frac{6}{2} = 3 \quad 3^2 = 9$$

$$\frac{5618}{2} = \frac{2(x + 3)^2}{2}$$

$$\sqrt{2809} = \sqrt{(x + 3)^2}$$

$$\pm 53 = x + 3$$

$$\begin{matrix} -3 \\ -3 \end{matrix}$$

$$W = 50 + 4 = 54 \text{ yd}$$

$$L = 2(50) + 4 = 104 \text{ yd}$$

Dimensions: 54 x 104 yds

$$\boxed{x = 50}$$

$$x = -56$$