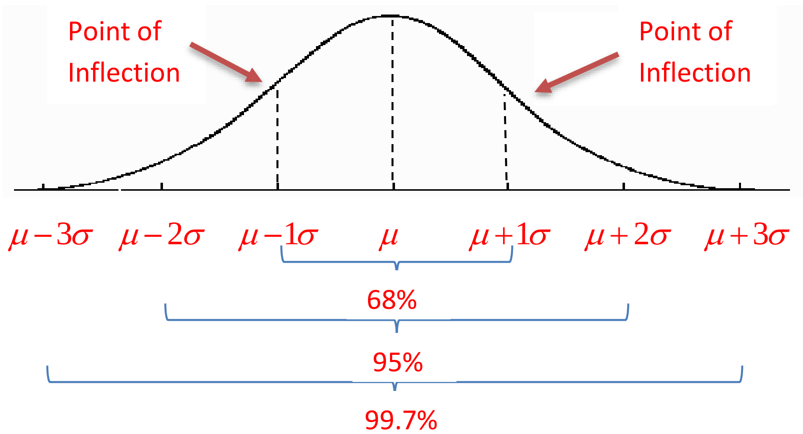
Name ____ Algebra 2

11.4: Normal Distributions – Guided Notes

Recall: A normal distribution is symmetric about the mean.

<u>Empirical Rule:</u>

- Approximately 68% of data will fall within 1 σ (1 standard deviation) of the mean
- Approximately 95% of data will fall within 2 σ (2 standard deviations) of the mean
- Approximately 99.7% of data will fall within 3 σ (3 standard deviations) of the mean



Notation:

$\overline{X}$ = mean (sample)

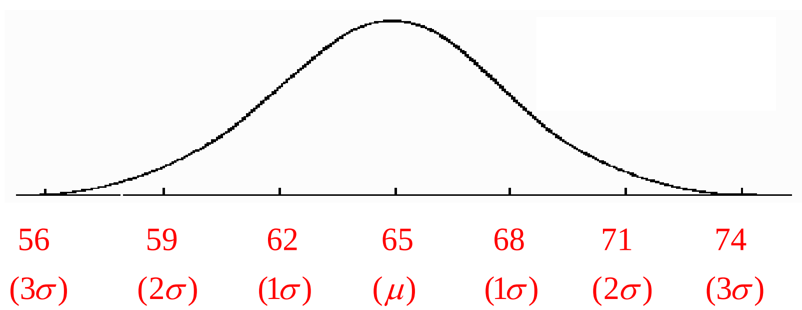
μ = mean (population)

S = standard deviation (sample)

σ = standard deviation (population)

<u>Ex1.</u> A sample of ISD high school students have their heights recorded. The average height was 65 inches with a standard deviation of 3 inches. Assume the data is normally distributed.

1. Sketch and label the normal curve:



2. What proportion (percent) of students are between 56 and 62 inches?

2.35%+13.5% = 15.85%

3. What would you expect to be the height of the shortest 2.5% of the population?

Less than 59 inches.

4. What would you expect to be the smallest and largest heights of the "middle" 95% of students?

Between 59 and 71 inches.

5. If 2,200 students were sampled, how many students are shorter than 56 inches? Taller than 74 inches?

0.15% * 2,200 students = 3.3 students Round down to 3 students.

Q. Do you know the upper and lower boundary values (the tallest and shortest person) for the whole population in this study?

No, the empirical rule covers 99.7% of the values and does not provide details about the remaining 0.3%.

Z–Score: the number of standard deviations away from the mean.

- $z = \dfrac{X-\mu}{\sigma}$, where μ is the population mean, σ is the population standard deviation, and X is the data-value.

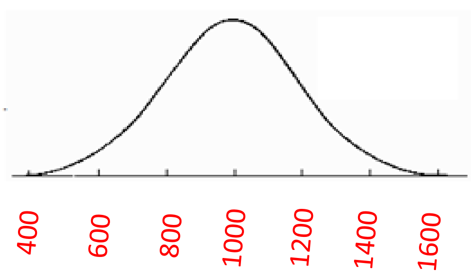
  **** Once you know the z-score, you can use a grapher or a table of values to find the corresponding percentage of the graph LOWER than the z-score. ****

Ex2. Ella and Alicia are comparing scores from their college entrance exams. Ella's SAT score is 1380. Alicia's ACT score is 32. The mean SAT score is 1000 with a standard deviation of 200. The mean ACT score is 21 with a standard deviation of 5. Assume the data is normally distributed.

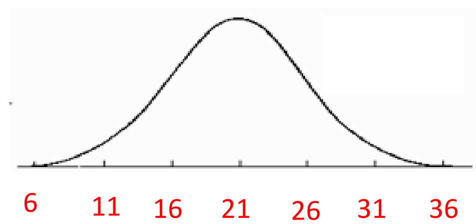
*** The scores cannot be compared directly, but we can compare their relative positions within each distribution. ***

1. Label the Normal Curves based on the information provided.

SAT:



ACT:



2. Calculate the z-scores for Ella and Alicia.

$$z = \frac{1380-1000}{200} = 1.9$$

3. April scored a 76 on the test; what is her z-score?

$$z = \frac{32-21}{5} = 2.2$$

4. Tim took the SAT and his result has a z-score of 0.8. What was his SAT score?

$$0.8 = \frac{X-1000}{200} \qquad X = 0.8(200) + 1{,}000 = 1{,}160$$

Percentile: the percentage of values less than or equal to a particular data value.

5. Jerry scored a 25 on the ACT. What is his percentile?

84th percentile

Standard Normal Distribution: a normal distribution with a mean of zero ($\mu = 0$) and a standard deviation of one ($\sigma = 1$). The total area under the curve of the standard normal distribution is equal to one (1), and represents 100% of the data values.