

Goal: Identify and Describe Arithmetic Sequences, identify terms in an arithmetic sequence, and find common difference. Write arithmetic sequences in recursive and explicit formulas.

A. There are 5 shaded squares in row 3. The total number of shaded squares up to and including row 3 is 9. Fill in the table for the remaining rows.

Row Number	1	2	3	4	5	6	8	10
Number of Shaded Squares in the Row	1	3	5	7	9	11	15	19
Total Number of Shaded Squares	1	4	9	16	25	36	64	100

What number patterns do you see in the rows of the table?

each row: previous row + 2
total shaded squares: $(\text{row } \#)^2$

$$a_1, a_2, a_3, a_4, a_5$$

EXAMPLE 1: Is the ordered list 26, 39, 52, 65, 78 an arithmetic sequence?

Yes! This is an arithmetic sequence with constant increase of 13.

Identify the first term: $a_1 = 26$

notation of 1st term of a sequence "a sub 1"

Identify the last term: $a_5 = 78$

5th term of sequence "a sub 5"

Find the common difference:

Common difference is "d" = +13 $d = 13$

How are sequences related to functions?

the Common difference IS like the Slope. It's a constant rate

Represent the sequence in subscript notation: The Recursive Formula of an Arithmetic Sequence:

$$a_n = a_{n-1} + d$$

the n^{th} term of the sequence previous term of the sequence common difference

i.e. $n=10$

$$a_{10} = a_{10-1} + d$$

$$a_{10} = a_9 + d$$

EXAMPLE 2

A. What is a recursive formula for the height above the ground of the n^{th} step of the pyramid shown?

$$d = 26$$

$$a_n = a_{n-1} + 26$$

*start with "d"

B. Use the recursive formula $a_n = a_{n-1} + 26$, where $a_1 = 26$, to find the height above the ground of the 3rd step.



Identify $a_1 = 26$

what we know

$$a_3 =$$

$$a_2 = 26 + 26$$

$$a_2 = 52 \text{ cm}$$

$$a_3 = a_{3-1} + 26$$

$$a_3 = a_2 + 26$$

$$a_3 = 52 + 26$$

$$a_3 = 78 \text{ cm}$$

The 3rd step is 78cm above ground

$$a_n = a_{n-1} + d$$

You try:

Given the arithmetic sequence

-4, -12, -20, -28, ...

$$\begin{array}{cccc} -4 & -12 & -20 & -28 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ -8 & -8 & -8 & -8 \end{array}$$

Write the recursive formula:

$$a_n = a_{n-1} - 8$$

You try: Given the arithmetic sequence

-6, -1, 4, 9, 14, ...

Write the recursive formula:

$$d = +5$$

$$\therefore a_n = a_{n-1} + 5$$

EXAMPLE 3

The cost of renting a bicycle is given in the table.

How can you represent the rental cost using an explicit formula?

Number of days rented (x)	1	2	3	4	5	10
Rental Cost (y)	26	38	50	62	74	134

The cost for renting a bicycle for 10 days is \$134.

The Explicit Formula

$$a_n = a_1 + (n-1)d$$

Common difference
(term number - one)
nth term 1st term

What is the cost of renting the bicycle for 10 days?

$$a_n = 26 + (n-1)12$$

$$n=10$$

$$a_{10} = 26 + (10-1)12$$

$$a_{10} = 26 + (9)12$$

$$a_{10} = 26 + 108$$

$$a_{10} = 134$$

How is the explicit formula of an arithmetic sequence related to a linear function?

$$a_n = 26 + 12n - 12 \Rightarrow a_n = 12n + 14$$

$$f(n) = 12n + 4$$

$$n=x$$

$$a_n = y$$

EXAMPLE 4

The recursive formula for the height above the ground of the n^{th} step of the stairs shown is $a_n = a_{n-1} + 4$ with $a_1 = 7$. What explicit formula finds the height above the ground of the n^{th} step?

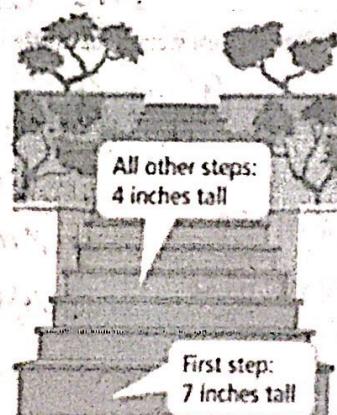
① Identify $a_1 = 7$ ② "sub" in to explicit formula
what we know $d = 4$ in to explicit formula
 $a_n = a_1 + (n-1)d$

$$a_n = 7 + (n-1)4$$

$$a_n = 7 + 4n - 4$$

③ Simplify explicit formula.

$$a_n = 4n + 3$$



EXAMPLE 5 Using the explicit formula

Given $a_n = 10 - 3(n-1)$

$$a_n = a_1 + d(n-1)$$

$$a_2 = 10 - 3(2-1)$$

$$a_2 = 10 - 3(1)$$

$$a_2 = 10 - 3$$

$$a_2 = 7$$

$$a_3 = 10 - 3(3-1)$$

$$a_3 = 10 - 3(2)$$

$$a_3 = 10 - 6$$

$$a_3 = 4$$

$$a_4 = 10 - 3(4-1)$$

$$a_4 = 10 - 3(3)$$

$$a_4 = 10 - 9$$

$$a_4 = 1$$

$$a_5 = 10 - 3(5-1)$$

$$a_5 = 10 - 3(4)$$

$$a_5 = 10 - 12$$

$$a_5 = -2$$

You try:

Given $a_n = 11 - 4(n-1)$

Find the first 5 terms of the sequence:

$$a_1 = 10$$

$$a_2 = 7$$

$$a_3 = 4$$

$$a_4 = 1$$

$$a_5 = -2$$

$$a_6 = -5$$

$$a_7 = -8$$

$$a_8 = -11$$

$$a_9 = -14$$

$$a_{10} = -17$$

$$a_{11} = -20$$

$$a_{12} = -23$$

$$a_{13} = -26$$

$$a_{14} = -29$$

$$a_{15} = -32$$

$$a_{16} = -35$$

$$a_{17} = -38$$

$$a_{18} = -41$$

$$a_{19} = -44$$

$$a_{20} = -47$$

$$a_{21} = -50$$

$$a_{22} = -53$$

$$a_{23} = -56$$

$$a_{24} = -59$$

$$a_{25} = -62$$

$$a_{26} = -65$$

$$a_{27} = -68$$

$$a_{28} = -71$$

$$a_{29} = -74$$

$$a_{30} = -77$$

$$a_{31} = -80$$

$$a_{32} = -83$$

$$a_{33} = -86$$

$$a_{34} = -89$$

$$a_{35} = -92$$

$$a_{36} = -95$$

$$a_{37} = -98$$

$$a_{38} = -101$$

$$a_{39} = -104$$

$$a_{40} = -107$$

$$a_{41} = -110$$

$$a_{42} = -113$$

$$a_{43} = -116$$

$$a_{44} = -119$$

$$a_{45} = -122$$

$$a_{46} = -125$$

$$a_{47} = -128$$

$$a_{48} = -131$$

$$a_{49} = -134$$

$$a_{50} = -137$$

$$a_{51} = -140$$

$$a_{52} = -143$$

$$a_{53} = -146$$

$$a_{54} = -149$$

$$a_{55} = -152$$

$$a_{56} = -155$$

$$a_{57} = -158$$

$$a_{58} = -161$$

$$a_{59} = -164$$

$$a_{60} = -167$$

$$a_{61} = -170$$

$$a_{62} = -173$$

$$a_{63} = -176$$

$$a_{64} = -179$$

$$a_{65} = -182$$

$$a_{66} = -185$$

$$a_{67} = -188$$

$$a_{68} = -191$$

$$a_{69} = -194$$

$$a_{70} = -197$$

$$a_{71} = -200$$

$$a_{72} = -203$$

$$a_{73} = -206$$

$$a_{74} = -209$$

$$a_{75} = -212$$

$$a_{76} = -215$$

$$a_{77} = -218$$

$$a_{78} = -221$$

$$a_{79} = -224$$

$$a_{80} = -227$$

$$a_{81} = -230$$

$$a_{82} = -233$$

$$a_{83} = -236$$

$$a_{84} = -239$$

$$a_{85} = -242$$

$$a_{86} = -245$$

$$a_{87} = -248$$

$$a_{88} = -251$$

$$a_{89} = -254$$

$$a_{90} = -257$$

$$a_{91} = -260$$

$$a_{92} = -263$$

$$a_{93} = -266$$

$$a_{94} = -269$$

$$a_{95} = -272$$

$$a_{96} = -275$$

$$a_{97} = -278$$

$$a_{98} = -281$$

$$a_{99} = -284$$

$$a_{100} = -287$$

$$a_{101} = -290$$

$$a_{102} = -293$$

$$a_{103} = -296$$

$$a_{104} = -299$$

$$a_{105} = -302$$

$$a_{106} = -305$$

$$a_{107} = -308$$

$$a_{108} = -311$$

$$a_{109} = -314$$

$$a_{110} = -317$$

$$a_{111} = -320$$

$$a_{112} = -323$$

$$a_{113} = -326$$

$$a_{114} = -329$$

$$a_{115} = -332$$

$$a_{116} = -335$$

$$a_{117} = -338$$

$$a_{118} = -341$$

$$a_{119} = -344$$

$$a_{120} = -347$$

$$a_{121} = -350$$

$$a_{122} = -353$$

$$a_{123} = -356$$

$$a_{124} = -359$$

$$a_{125} = -362$$

$$a_{126} = -365$$

$$a_{127} = -368$$

$$a_{128} = -371$$

$$a_{129} = -374$$

$$a_{130} = -377$$

$$a_{131} = -380$$

$$a_{132} = -383$$

$$a_{133} = -386$$

$$a_{134} = -389$$

$$a_{135} = -392$$

$$a_{136} = -395$$

$$a_{137} = -398$$

$$a_{138} = -401$$

$$a_{139} = -404$$

$$a_{140} = -407$$

$$a_{141} = -410$$

$$a_{142} = -413$$

$$a_{143} = -416$$

$$a_{144} = -419$$

$$a_{145} = -422$$

$$a_{146} = -425$$

$$a_{147} = -428$$

$$a_{148} = -431$$

$$a_{149} = -434$$

$$a_{150} = -437$$