8.0 Reduced Radical Form and Operations with Radicals

Multiplying and Dividing Radicals: We can separate and/or combine square roots when multiplying and dividing.

Examples:

1. $\sqrt{9}\cdot\sqrt{8}$

2. $\frac{\sqrt{54}}{\sqrt{2}}$

3. $\left(\sqrt{12}\right)\left(\sqrt{2}\right)$

Reduced Radical Form: All roots of perfect powers are taken out of the radical and written in front <u>AND</u> no radicals are in the denominator.

We need to recognize perfect squares in order to put square roots in reduced radical form:

1	2	3	4	5	6	7	8	9	10	11	12

Examples: Write the numbers below in reduced radical form.

1. $\sqrt{72}$

2. $\sqrt{27}$

3. $\sqrt{24}$

4. $\frac{2}{\sqrt{3}}$

5. $\frac{4}{\sqrt{8}}$

6. $\sqrt{\frac{32}{20}}$

Adding and Subtracting Radicals: We can only add/subtract radicals with the same <u>index</u> and same <u>radicand</u>. We want to put radicals in reduced radical form so we know what we can and cannot combine.

Examples:

1. $2\sqrt{2}+3\sqrt{8}$

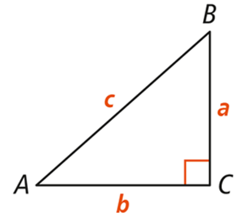
2. $5\sqrt{3}-2\sqrt{27}+\sqrt{6}$

3. $\sqrt{5}+\sqrt{2}-2\sqrt{20}+\sqrt{18}$

8.1 Right Triangles and the Pythagorean Theorem

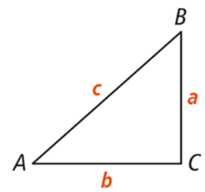
Thm 8-1 Pythagorean Theorem: If a triangle is a right triangle, then the sum of the squares of the legs is equal to the square of the hypotenuse.

If... $\triangle ABC$ is a right triangle **Then...**



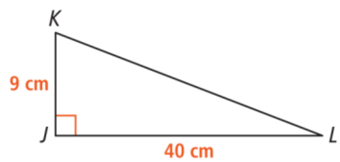
Thm 8-2 Converse of the Pythagorean Theorem: If the sum of the squares of two sides of a triange is equal to the square of the third side, then the triangle is a right triangle.

If... $a^2 + b^2 = c^2$ **Then...**

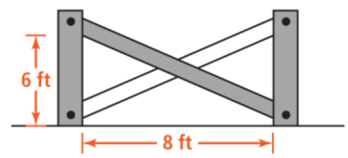


Example 1: Use theorems 8-1 and 8-2 to solve the problems below.

1. Find *KL*.



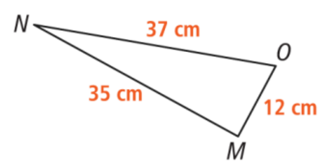
2. Why would a rancher build a fence and crossbeams with the measurements shown?



You Try!

1. To satisfy safety regulations, the distance from the wall to the base of a ladder should be at least one-fourth the length of the ladder. Did Drew set up the ladder correctly?

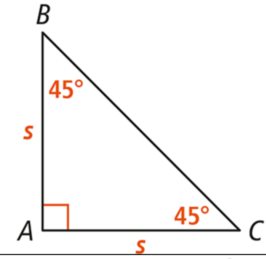
2. Is ΔMNO a right triangle? Justify algebraically.



Special Right Triangles

Thm 8-3 45° -45° -90° Triangle Theorem: In a 45° -45° -90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

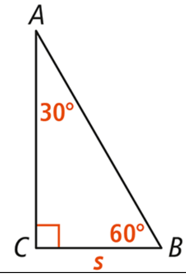
If...



Then...

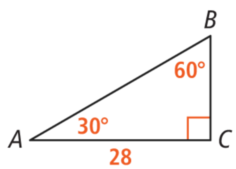
Thm 8-4 30° -60° -90° Triangle Theorem: In a 30° -60° -90° triangle, the length of the hypotenuse is 2 times the length of the short leg. The length of the long leg is $\sqrt{3}$ times the length of the short leg.

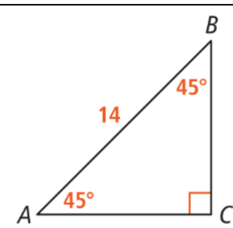
If...



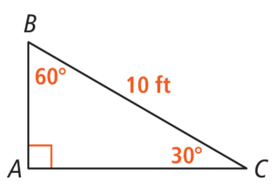
Then...

Example 2: Solve for the missing side lengths in both triangles below.

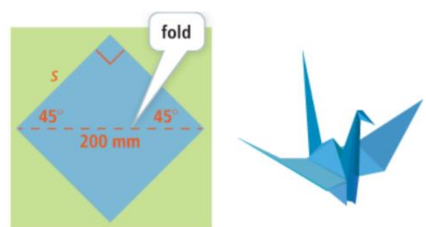




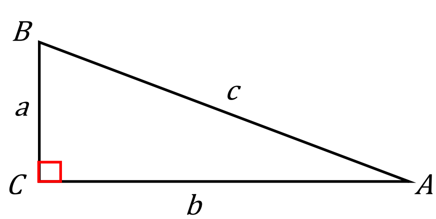
Example 3: Shawn only has 12 feet of board to make the horizontal and vertical supports. Can he do it?



Example 4: Find the side length and area of the paper square used for origami.



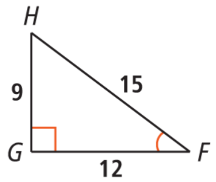
8.2 Trigonometric Ratios



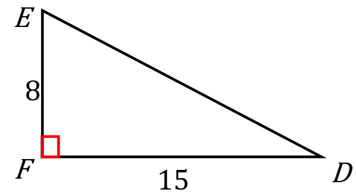
Trig Function	Abbreviation and Notation	Example of Ratio
Sine		
Cosine		
Tangent		

Example 1: Write all three trig ratios for both of the acute angles below.

A.

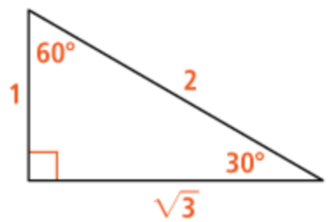


B.

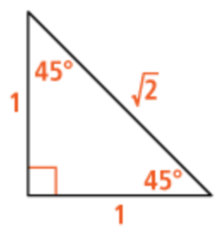


Example 2: Write all three trig ratios for each acute angle in the special right triangles below. Remember to rationalize the denominator!

A.



B.



Key Concept: Right triangles with one congruent angle will be similar by AA~. Due to proportions, the trig ratios for an angle measure produce the same ratio regarless of triangle size!

Calculator Use: Your calculator can switch between degrees and radians. For this topic, all angles will be measured in degrees. Be sure your calculator mode indicates degrees!

Example 3: A plane takes off and climbs at a 12° angle. Is that angle sufficient enough to fly over an 11,088-foot mountain that is 12.5 miles from the runway or does the plane need to increase its angle of ascent?

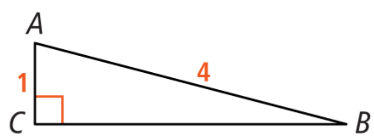
Inverse Trigonometric Functions

Inverse Function: A function that undoes the action of another function. For example, "squaring" and "square root" undo each other. The outputs and inputs generally switch in inverse functions.

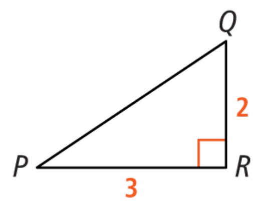
Trigonometric Function	Inverse Trigonometric Function
Input: Output: Notation:	Input: Output: Notation:

Example 4:

A. Find $m\angle A$. Round to the nearest hundredth.



B. You try! Find $m\angle P$ and $m\angle Q$ to two decimals.



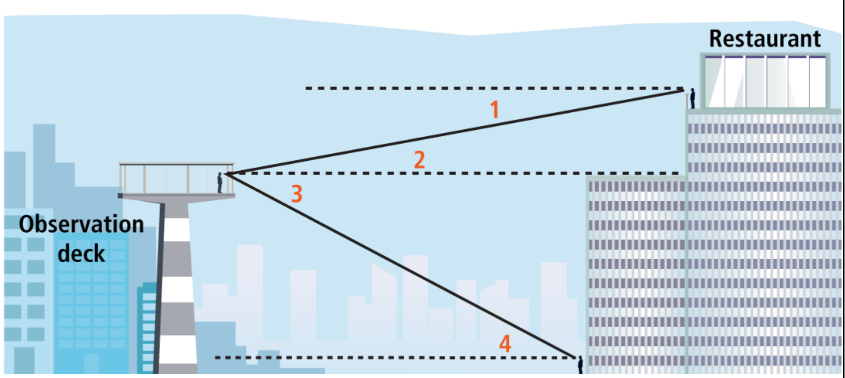
8.5 Problem Solving with Trigonometry

Angle of Elevation:

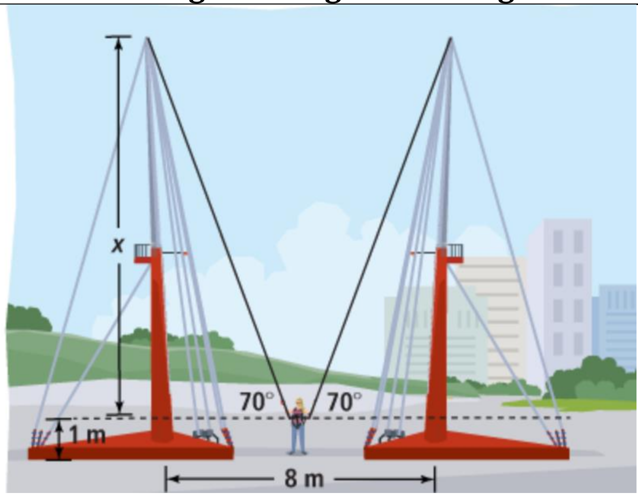
Angle of Depression:

Identifying Angles of Elevation and Angles of Depression

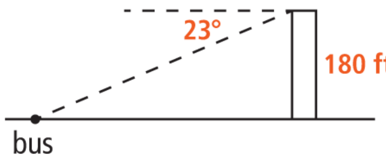
Locate ∠2 and ∠3 in the diagram, and classify each as an angle of elevation or an angle of depression. Justify your reasoning.



Example 1: Reagan stands between two bungee poles for a reverse bungee ride. Two of the bungee cords from the top of the posts extend down to her waist, connecting to a harness 1 m above the ground. The poles are 8 m apart, and the cords form a 70° angle with the horizon. How tall are the poles?



You Try! Nadeem sees the tour bus from the top of the tower. To the nearest foot, how far is the bus from the base of the tower?



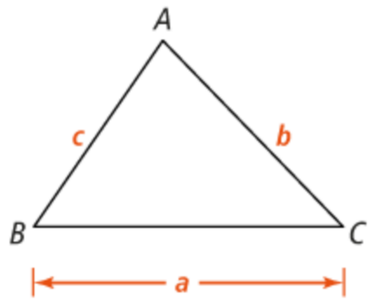
8.3 The Law of Sines

Example 1: Developing the Law of Sines

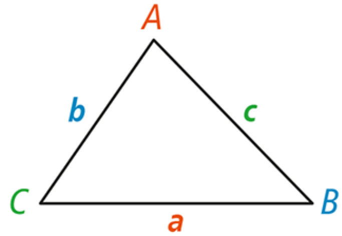
Step 1: construct the altitude to A; label it h

Step 2: write trig ratios for $\angle B$ and $\angle C$

Step 3: use substitution to write a proportion

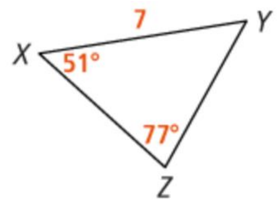


Law of Sines: For any ΔABC with side lengths a, b, and c for angles A, B, and C, respectively, the Law of Sines relates the sine of each angle to its opposite side.

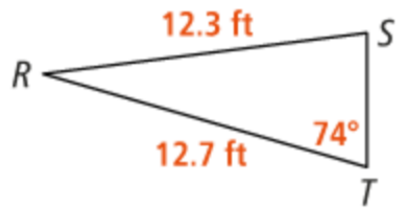


Works for non-right triangles!

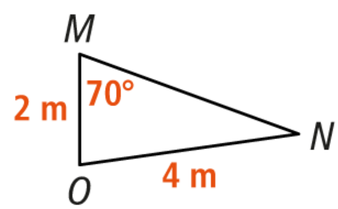
Example 2: Find XZ and YZ to the nearest hundredth.



Example 3: Find $m\angle R$ and $m\angle S$ in the triangle below. Round to the nearest hundredth.

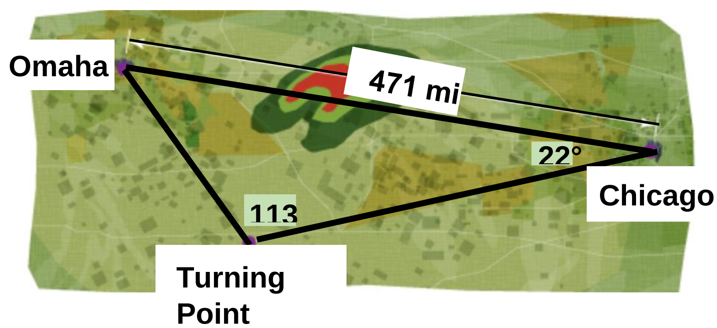


You Try! Find $m\angle N$ and $m\angle O$ in the triangle below. Round to the nearest hundredth.



Ambiguous Case: When do you get two possible answers when using the Law of Sines?

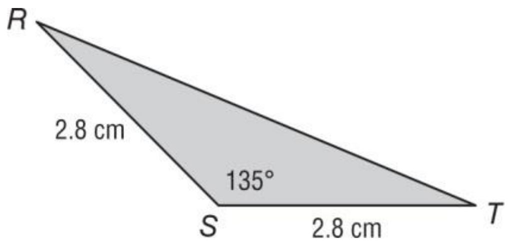
Example 4: Application. The map shows the path a pilot flew between Omaha and Chicago in order to avoid a thunderstorm. How much longer is this route than the direct route to Chicago?



Area of a Triangle: We can use the sine function to find the area of any triangle using the following formula:

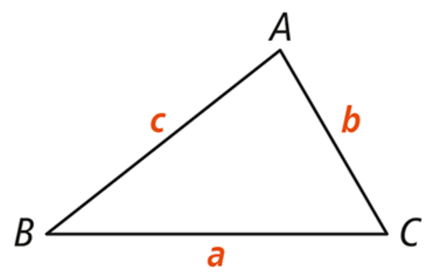
$$A_{\Delta} = \frac{1}{2}ab \cdot \sin C$$

Using the sine ratio of the angle between the two known sides creates a height for the triangle. Try it out by finding the area of ΔRST :



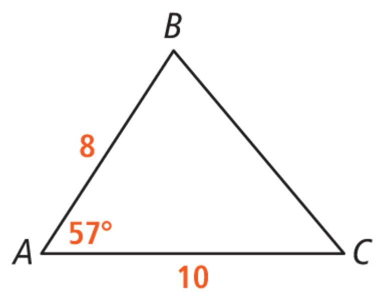
8.4 The Law of Cosines

Law of Cosines: For any ΔABC with side lengths a, b, and c for angles A, B, and C, respectively, the Law of Cosines relates the cosine of each angle to the other sides of the triangle.



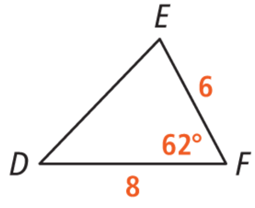
Notice Patterns: What patterns can you see to help you memorize these formulas?

Example 1: What is BC to the nearest hundredth? *You need to show what you are plugging in on your paper!*

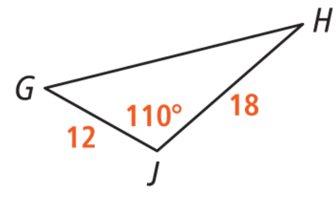


You Try!

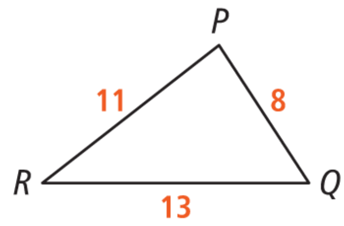
A. Find DE to two decimal places.



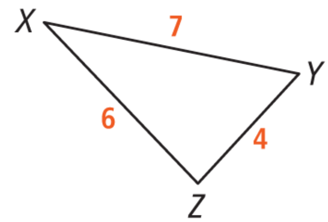
B. Find GH to the nearest hundredth.



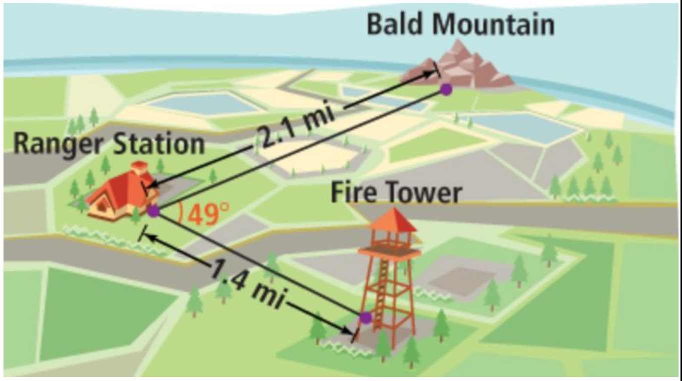
Example 2: Using LoC to find an Angle. To the nearest hundredth, what is $m\angle P$?



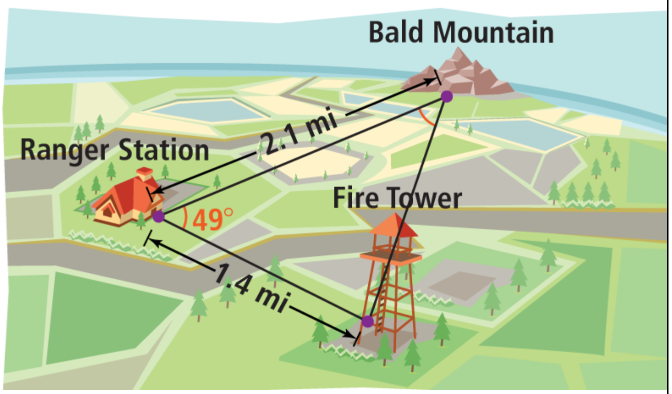
You Try! To the nearest hundredth, what is $m\angle X$?



Example 3: Application. The district ranger wants to build a new ranger station at the location of the fire tower because it would be closer to Bald Mountain than the old station is. Is the district ranger correct? Explain.



You Try! Assume a path is drawn from the fire tower to Bald Mountain. What is the angle the new path forms with the old path from Bald Mountain to the ranger station?



Challenge Problems!

1. A 60 foot ramp makes an angle of 4.8° with the ground. To meet new accessibility guidelines, the ramp must make an angle of 2.9° with the ground.

a. How long will the new ramp be?

b. How much farther along the ground will the new ramp extend?

2. In a ship sailing north, a woman notices that a hotel on a shore has a bearing of N 20^{0} E. A little while later, after having sailed 40 km, she observes that the bearing of the hotel is now S 80^{0} E. How far is the ship from the hotel at the time of the second sighting?