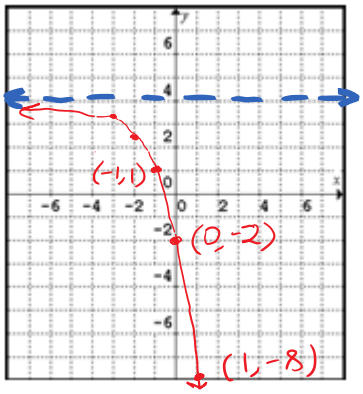
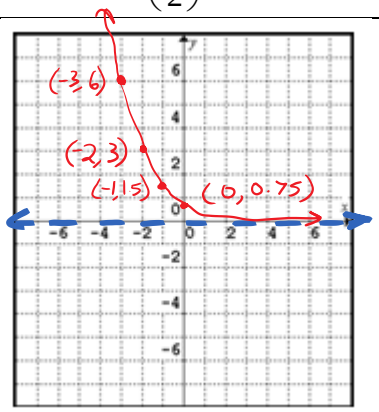
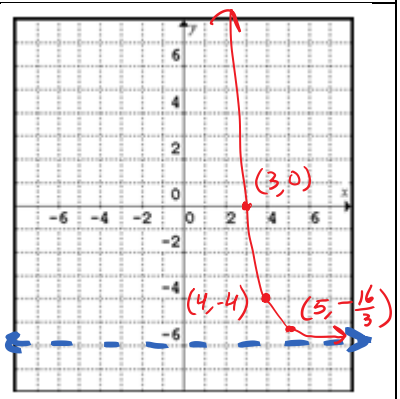
Name: Key **Class Period:** _____ **Date:** _____

Algebra 2 **Topic 6 Review WS**

1. Sketch each graph, showing *at least* 3 points and its asymptote. Then answer the questions. ***NC***

a. $f(x) = -3 \cdot 2^{x+1} + 4$	**b.** $g(x) = 6\left(\frac{1}{2}\right)^{x+3}$	**c.** $h(x) = 2 \cdot 3^{-(x-4)} - 6$
		
Transformations: Reflection across x-axis V. Dilation by s.f. of 3 H. translation left 1 V. translation up 4	**Transformations:** Vertical dilation by s.f. of 6 Horizontal translation left 3	**Transformations:** Reflection across y-axis Vertical dilation by s.f. of 2 H. translation right 4 V. translation down 6
Domain: $x \in \mathbb{R}$	***y*-intercept:** $\left(0, \frac{3}{4}\right)$	***x*-intervals where** $h(x) > 0$ $x \in (-\infty, 3)$
Range: $f(x) \in (-\infty, 4)$	**Range:** $g(x) \in (0, \infty)$	***x*-intervals where** $h(x) < 0$ $x \in (3, \infty)$
***y*-intercept:** $(0, -2)$	**End Behavior:** as $x \to \infty$, $g(x) \to 0^+$ as $x \to -\infty$, $g(x) \to \infty$	**Asymptote:** $y = -6$

2. Multiple Choice. Which of the following models an account value, $A(t)$, after t years where the annual percent rate (APR) is 6.7%, the principal is \$3,050, and the account is compounded quarterly?

[A] $A(t) = 3{,}050(1 + 0.067)^{4t}$

[B] $A(t) = 3{,}050\left(1 + \frac{0.67}{4}\right)^{4t}$

[C] $A(t) = 3{,}050\left(1 + \frac{0.067}{4}\right)^{t}$

[D] $A(t) = 3{,}050\left(1 + \frac{0.067}{4}\right)^{4t}$

3. Assume that 2,200 students attended IHS in 2014 and 2,450 students attended IHS in 2017. Write the equation of a function $f(x) = a \cdot b^x$ that models the number of students attending IHS, $f(x)$, x years since 2010. Use your equation to find the growth rate, expressed as a percent and rounded to 3 decimals.

$(4, 2{,}200)$ & $(7, 2{,}450)$

$2{,}200 = ab^4$

$a = \frac{2200}{b^4}$

$2450 = \left(\frac{2200}{b^4}\right) b^7$

$b^3 = \frac{2450}{2200}$

$b = \left(\frac{2450}{2200}\right)^{1/3}$

$b \approx 1.036528229\ldots$

$a \approx 1905.8913\ldots$

$\therefore f(x) = 1905.891(1.037)^x$

$b = 1 + r \rightarrow r = b - 1$

$\therefore r \approx 0.036528\ldots$

$\therefore r \approx 3.653\%$

4. Evaluate each logarithm. ***NC***

a. $\log_8 \frac{1}{64}$

$8^{\square} = \frac{1}{64}$

-2

b. $\log_{125} 5$

$125^{\square} = 5$

$\frac{1}{3}$

c. $\log 1{,}000{,}000$

$10^{\square} = 1{,}000{,}000$

6

d. $\log_7(-7)$

$7^{\square} = -7$

DNE

e. $\log_2 1$

$2^{\square} = 1$

0

d. $\log_9 9^{12}$

$9^{\square} = 9^{12}$

12

5. Find the value of each logarithm rounded to three decimal places. ***C***

a. $\log_3 85$

4.044

b. $\log_{36} 5$

0.449

c. $\log_{4.2} 0$

DNE

6. Solve the equations below, expressing your answer as a *simplified fraction* or *integer*. ***NC***

a. $27^{3x+1} = 81^{x-3}$

$(3^3)^{3x+1} = (3^4)^{x-3}$

$3^{9x+3} = 3^{4x-12}$

$9x + 3 = 4x - 12$

$5x = -15$

$x = -3$

b. $3\log\left(\frac{4}{x}\right) = 6$

$\log\left(\frac{4}{x}\right) = 2$

$10^2 = \frac{4}{x}$

$x = \frac{4}{100}$

$x = \frac{1}{25}$

c. $\log_4 2 + 8x = \log_2 1 - 4.5$

$\frac{1}{2} + 8x = 0 - 4.5$

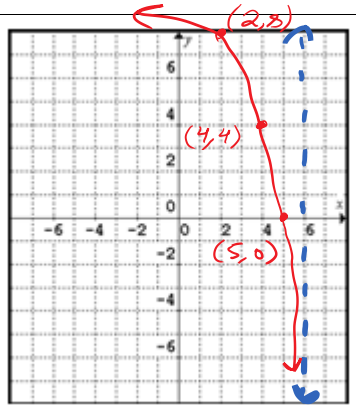
$8x = -4.5 - 0.5$

$8x = -5$

$x = -\frac{5}{8}$

7. Graph the logarithms, showing *at least* two points and its asymptote. Then answer the questions. ***NC***

a. $f(x) = 4\log_2(-(x-6))$



End Behavior:

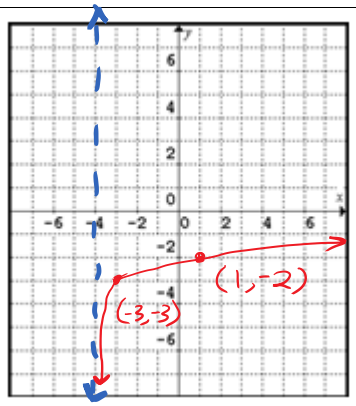
as $x \to 6^-$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to \infty$

Transformations:

Reflection across y-axis,
V. Dilation by s.f. of 4
H. Translation right 6

b. $g(x) = \log_5(x+4) - 3$



End Behavior:

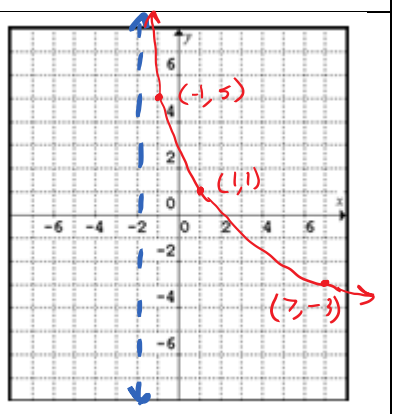
as $x \to \infty$, $g(x) \to \infty$

as $x \to -4^+$, $g(x) \to -\infty$

Transformations:

H. translation left 4
V. translation down 3

c. $h(x) = -4\log_3(x+2) + 5$



End Behavior:

as $x \to \infty$, $h(x) \to -\infty$

as $x \to -2^+$, $h(x) \to \infty$

Transformations:

Reflection across x-axis
V. dilation by s.f. of 4
H. translation left 2
V. translation up 5

8. The function $y = g(x)$ is a transformation of $f(x) = 4^x$. ***NC***

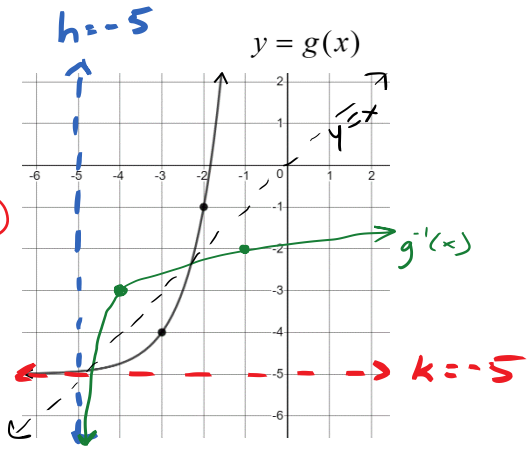
a. Assuming only translations were applied, write the equation of $g(x)$.

$g(x) = 4^{x+3} - 5$

(use $k=-5$ as asymptote & $(-3,-4)$ as start point)

b. Using properties of inverses, graph $g^{-1}(x)$ on the grid to the right.

c. Using your answer from part (a), find the equation for $g^{-1}(x)$. Verify your equation is accurate by testing points from part (b) into your equation.



$y = 4^{x+3} - 5$

$4^{x+3} = y + 5$

$\log_4(y+5) = x+3$ → $x = \log_4(y+5) - 3$

$\therefore g^{-1}(x) = \log_4(x+5) - 3$

test $(-4,-3)$ → $g^{-1}(-4) = \log_4(1) - 3 = 0 - 3 = -3$ ✓

9. Compute the value of each sum below. In questions (b) and (c), classify as convergent or divergent. ***C***

a. $\sum_{n=1}^{10} 4(1.3)^{n-1}$

$S_{10} = \frac{4(1-1.3^{10})}{1-1.3}$

$S_{10} \approx 170.478$

b. $\sum_{n=1}^{\infty} -2\left(\frac{3}{5}\right)^{n-1}$ Convergent $|r|<1$

$S = \frac{-2}{1-\frac{3}{5}}$

$S = -2 \cdot \frac{5}{2}$

$S = -5$

c. $\sum_{n=1}^{\infty} 5\left(\frac{7}{4}\right)^{n-1}$ Divergent

The sum DNE since $r = \frac{7}{4}$ & $|r| > 1$.

10. The half-life of carbon-14 is 5,730 years. This is the amount of time it takes for half of a sample to decay. From a sample of 24 grams of carbon-14, how long will it take until only 3 grams of the sample remain? ***C***

* Need to know how many half-lives occurred, then multiply by length of half-life. *

Explicit formula: $a_n = 24\left(\frac{1}{2}\right)^n$; $n \geq 0$

(24 is a_0 since no half-lives occurred yet)

$a_n = 3$

$3 = 24\left(\frac{1}{2}\right)^n$

$\frac{1}{8} = \left(\frac{1}{2}\right)^n$

$\therefore n = 3$ (3 half-lives occurred)

$3(5,730) = 17,190$

$\therefore$ It takes 17,190 yrs

11. How many terms are in the geometric series $2.1 + 10.5 + \ldots + 820,312.5$? ***C***

$a_1 = 2.1$; $r = 5$ → $a_n = 2.1(5)^{n-1}$, $n \geq 1$ (×5)

$820,312.5 = 2.1(5)^{n-1}$

$5^{n-1} = 390,625$

$\log_5 390,625 = n-1$

$8 = n-1$

$n = 9$

$\therefore$ there are 9 terms total.

12. A hurricane center uses the function $s = 95\log d + 75$ to relate the wind speed in miles per hour, s, and the distance in miles a hurricane travels, d. How many miles will a hurricane travel with a wind speed of approximately 320 mph? ***C***

$320 = 95\log d + 75$

$-75 \quad -75$

$\frac{245}{95} = \frac{95\log d}{95}$

$\log d = \frac{245}{95}$

$10^{\left(\frac{245}{95}\right)} = d$

$d \approx 379.269$ miles

Additional Application Problems

13. Darren wants to invest $4,500 into an account that earns 5% annual interest. Help him see how much each account below would earn after 10 years if it is compounded according to the period listed. ***C***

Compounding Period	Use the Compound Interest Formula	Account Value
Annually	$A(t) = 4500\left(1 + \frac{0.05}{1}\right)^{1t}$	$A(10) \approx \$7330.026$
Quarterly	$A(t) = 4500\left(1 + \frac{0.05}{4}\right)^{4t}$	$A(10) \approx \$7396.288$
Monthly	$A(t) = 4500\left(1 + \frac{0.05}{12}\right)^{12t}$	$A(10) \approx \$7411.543$
Daily	$A(t) = 4500\left(1 + \frac{0.05}{365}\right)^{365t}$	$A(10) \approx \$7418.992$

14. A professor was interested in the relationship between time and memory. The professor determined the model $f(t) = t_0 - 15\log(t+1.1)$ gives the memory score after t months when a student had an initial memory score of t_0. ***C***

a. Write a model for a student with an initial memory score of 95. $t_0 = 95$

$\therefore f(t) = 95 - 15\log(t+1.1)$

b. After how many *years* will the student from part (a) have a memory score of 65? Round to the nearest year.

$f(t) = 65$

$\therefore 65 = 95 - 15\log(t+1.1)$

$-30 = -15\log(t+1.1)$

$\log(t+1.1) = 2$

$10^2 = t + 1.1$

$t = 98.9$ months! need years

$\frac{98.9}{12} \approx 8.2416\ldots$

$\therefore$ The student's memory score is 65 after about 8 years.

15. The pH of a solution is a measure of its concentration of hydrogen ions. This concentration, written as $[H^+]$ and measured in moles per liter, is given by the formula $pH = \log\frac{1}{[H^+]}$. What is the concentration of hydrogen ions in a liter of vinegar that has a pH level of 2.5? ***C***

$2.5 = \log\frac{1}{[H^+]}$

$10^{2.5} = \frac{1}{[H^+]}$

$[H^+] = \frac{1}{10^{2.5}}$

$\therefore$ The concentration of H^+ is $10^{-2.5}$ or ≈ 0.003 moles per liter.

16. Mateo invested $12,000 into an account that earns 4.5% compounded quarterly. If he leaves the account untouched, during which year (since initially investing) will the account double in value? ***C***

$\$24{,}000 = \$12{,}600\left(1 + \frac{0.045}{4}\right)^{4t}$

$2 = \left(1 + \frac{0.045}{4}\right)^{4t}$

$\log_{\left(1+\frac{0.045}{4}\right)}(2) = 4t$

$t = \frac{\log_{\left(1+\frac{0.045}{4}\right)}(2)}{4}$

$t \approx 15.4897\ldots$

$\therefore$ During the 15th year, the account value doubles.