$S_n = \frac{n}{2}(a_1 + a_n)$

6.7 Geometric Sequences and Series – Day 1

Warm up! Express the series below in sigma notation, then find the sum.

4 + 7 + 10 + ... + 25

$a_n = 4 + 3(n-1) \quad n \geq 1$

$25 = 4 + 3(n-1)$

$21 = 3(n-1)$

$7 = n-1$

$n = 8$

$\sum_{n=1}^{8} 4 + 3(n-1)$

$S_8 = \frac{8(4+25)}{2} = 116$

Geometric Sequence:

- a sequence with a constant ratio between terms.
- multiply by a constant value each term. This value is the common ratio, r

Recursive Definition for Geometric Sequences:

$$a_n = \begin{cases} a_1 & n = 1 \\ a_{n-1} \cdot r & n > 1 \end{cases}$$

Ex. 1: Decide if the sequences below are geometric. If so, write the recursive definition.

A. 1.22, 1.45, 1.68, 1.91, ...

$\frac{1.45}{1.22} \quad \frac{1.68}{1.45}$

$1.19 \quad 1.16$

Not geometric

B. −1.5, 0.75, −0.375, 0.1875, ...

$\frac{.75}{-1.5} \quad \frac{-.375}{.75} \quad \frac{.1875}{-.375}$

$-\frac{1}{2} \quad -\frac{1}{2} \quad = \frac{1}{2}$

Geometric

$$a_n = \begin{cases} -1.5 & n = 1 \\ a_{n-1} \cdot \left(-\frac{1}{2}\right) & n > 1 \end{cases}$$

Ex. 2: Given the recursive definition below, what is an explicit definition for the sequence? *Hint: look for a pattern!*

$$a_n = \begin{cases} 5, & n = 1 \\ \frac{1}{2} a_{n-1}, & n > 1 \end{cases}$$

$a_1 = 5$

$a_2 = \left(\frac{1}{2}\right)(5) = 2.5$

$a_3 = \left(\frac{1}{2}\right)(2.5) = 1.25$

$a_4 = \left(\frac{1}{2}\right)(1.25) = .625$

each time, take the previous term & multiply by the common ratio

Explicit Definition for Geometric Sequences:

$a_n = a_1 \cdot r^{n-1} \quad n \geq 1$

Ex. 3: A phone tree is when one person calls a certain number of people, then those people each call the same number of people, and so on. In the first round of a phone tree, three people were called. In the fifth round of calls, 243 people were called.

A. Write an explicit definition to find the number called in each round.

$a_1 = 3 \quad a_5 = 243 \quad r = 3$

$243 = 3r^{5-1}$

$81 = r^4$

$a_n = 3(3)^{n-1} \quad n \geq 1$

B. How many people were called in the eighth round of the phone tree?

$a_8 = 6561$ were called

6.7 Geometric Sequences and Series – Day 2

Geometric Series:

Finite vs. Infinite:

finite — a series that ends (doesn't go to infinity)

infinite — goes on forever

Convergent vs. Divergent

Convergent: a series with a sum

Divergent: a series that doesn't have a sum (b/c adding values that are app ∞)

Ex. 1: Classify each series or situation below as convergent (has a sum) or divergent (doesn't have a sum).

A. $2+4+8+16+...+256$ convergent, finite # of terms added

B. $-3+9-27+81-243+...$

... implies goes on forever

Divergent: infinite # of terms that keep getting larger

C. Your parents give you \$50 for your birthday, then \$25 for your next birthday, then \$12.50 for the birthday after that, and so on.

Convergent: despite implying infinite, the values get smaller

Sum of Finite Geometric Series	Sum of Infinite Geometric Series
$S_n = \frac{a_1(1-r^n)}{1-r}$ $r \neq 1$ a_1: 1st term value, r: common ratio n: # of terms	$S = \frac{a_1}{1-r}$, $-1 < r < 1$

Ex. 2: Finite Geometric Series.

A. Express $3+6+12+...+768$ in sigma notation, then find the sum.

$768 = 3(2)^{n-1}$

$256 = 2^{n-1}$

$\log_2 256 = n-1$

$8 = n-1$

$9 = n$

$$\sum_{n=1}^{9} 3(2)^{n-1} \quad n \geq 1$$

$$S_9 = \frac{3(1-2^9)}{1-2}$$

$$S_9 = 1533$$

B. The sum of a geometric series is 155. The first term of the series is 5, and its common ratio is 2. How many terms are in the series?

$$155 = \frac{5(1-2^n)}{1-2}$$

$$-155 = 5(1-2^n)$$

$$-31 = 1-2^n$$

$$32 = 2^n$$

$$\boxed{5 = n}$$

5 terms

Ex. 3: Infinite Geometric Series. Write in sigma notation (if not already in it), then find the sum if it exists.

A. 0.2 + 0.02 + 0.002 + 0.0002 +…

$\sum_{n=1}^{\infty} 2(0.1)^{n-1}$, $n \geq 1$ $S = \frac{0.2}{1-0.1} = \frac{0.2}{0.9} = \frac{2}{9}$ or 0.222

B. 1 + 10 + 100 +1000 + …

$\sum_{n=1}^{1} 1(10)^{n-1}$, $n \geq 1$ $r > 1$ so sum doesnt exist b/c series diverges

C. $\sum_{n=1}^{\infty} 1{,}024\left(\frac{1}{4}\right)^n$

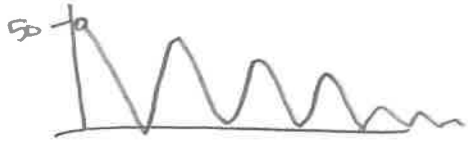
$S = \frac{256}{1-1/4} = \frac{256}{3/4} = 341\,1/3$

$a_1 = 1024\left(\frac{1}{4}\right)^1$

$= 256$

Ex. 4: A ball was dropped from 50cm. The ball rebounds to a height of 40cm, then 32cm, then 25.6cm and so on. What is the total vertical distance traveled?

$\frac{40}{50} = \frac{4}{5} = 0.8$



$S_{Down} = \frac{50}{1-0.8} = 250$

total dist: 450 cm

$S_{up} = \frac{40}{1-0.8} = 200$

Ex 5: Additional Practice. Write in Sigma notation and find the sum (if possible).

A. 75 + 37.5 + 18.75 +…

$\sum_{n=1}^{\infty} 75\left(\frac{1}{2}\right)^{n-1}$ $n \geq 1$ $S = \frac{75}{1-1/2} = 150$

B. 5 + 10 + 20 + 40 +…

$\sum_{n=1}^{\infty} 5(2)^{n-1}$ Sum DNE, series diverges

C. 18 + 30 + 42 + 54 + 66

+12 +12 +12 +12

Arithmetic Series

$\sum_{n=1}^{5} 18+12(n-1)$ $n \geq 1$ $S = \frac{5}{2}(18+66) = 210$

D. −4 −12 −36 −… −8748

•3 •3

$\sum_{n=1}^{8} -4(3)^{n-1}$ $n \geq 1$ $S = \frac{-4(1-3^8)}{1-3}$

$= -13120$

$-8748 = -4(3)^{n-1}$

$2187 = 3^{n-1}$

$7 = n-1$ $n=8$