Name Key Date _____ Period _____

Algebra 2: Topic 11 Midterm Review

Match the following statements to the word that fills in the blank:

1. Bias is a systemic error in a study caused by the sampling method.
2. With cluster sampling a population is divided into groups and entire groups are chosen as the sample.
3. In a(n) survey all members of the sample are asked the same set of questions.
4. A(n) self-selected sample is made up of only volunteers.
5. A parameter is a measure that describes the population.

a. cluster
b. survey
c. parameter
d. bias
e. self-selected

6. Identify the following as a *statistic* or *parameter*.
   a. A National study was conducted of *all* Americans found that the average American drinks 2.1 coffee drinks a day. parameter
   b. The average Math SAT score of 100 *selected juniors* is 580. statistic
   c. The average height of *all* players in the NBA is 6 feet 7 inches. parameter

7. Mrs. McCormick wants to know what electives students want added next year. During 3rd period, she selects one class from each department (math, science, art, history, PE, language arts, etc.) and surveys all students within the selected classes.
   a. What is the sample in this situation?
   One 3rd period class from each department
   b. What is the population?
   All students at I.H.S.
   c. What might be a *statistic* Mrs. McCormick can gather from this data?
   Top 3 electives of surveyed students

Multiple Choice (1):

d. V. Select the sampling method used by Mrs. McCormick.

i.	Self-Selected	iv.	Cluster
ii.	Convenience	(v.)	Stratified
iii.	Systematic		

e. What may be a source of bias introduced by using this method? Answers may vary.
Not all departments represented equally during 3rd period. BC students not on campus during 3rd

8. What kind of study you would conduct to answer each statistical question?
   a. "What time of day are birds more active around the bird feeders?"
   Observational.
   b. "The garden club has planted a vegetable garden at the school. On what side of the school does the garden produce the highest yield?"
   Experimental

9, 11, 13, 15, 17, 17, 19, 19, 21, 23, 25, 27.

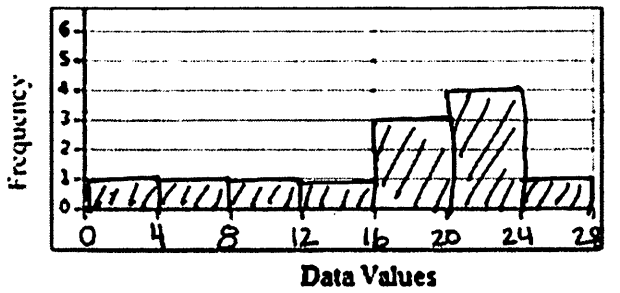
For 9-10, use the given data set to answer the questions below.

9. 3, 6, 12, 14, 17, 17, 18, 21, 21, 22, 23, 28

Min	Q1	Median	Q3	Max
3	13	17.5	21.5	28

IQR: 8.5 Mean: $\frac{202}{12} = 16.833$

Draw a Histogram:



Describe the data spread as skewed left, skewed right, or symmetric: Skewed left

Best Measure of Center: Median

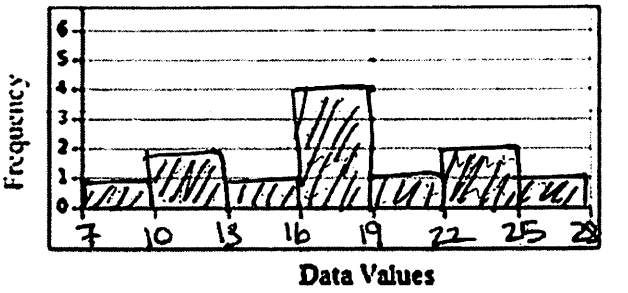
Best Measure of Spread: IQR

10. 17, 9, 27, 13, 15, 19, 19, 21, 11, 23, 17, 25

Min	Q1	Median	Q3	Max
9	14	18	22	27

IQR: 8 Mean: $\frac{216}{12} = 18$

Draw a Histogram:



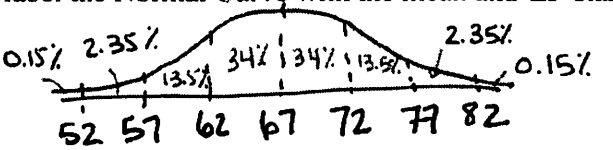
Describe the data spread as skewed left, skewed right, or symmetric: Symmetric

Best Measure of Center: Mean

Best Measure of Spread: Standard deviation

11. The mean of the summer average temperature recorded at all western regional airports is 67 degrees Fahrenheit with a standard deviation of 5 degrees F. Assume the airport temp data is normally distributed.

a. Sketch and label the Normal Curve with the mean and ±3 standard deviations based on the information provided.



b. What percentage of airports have a summer average temperature between 57 and 72 degrees F? **b.** 81.5%

c. Honolulu's airport temperature is higher than 97.5% of the airports in the region, what is the temperature? **c.** > 77°F

d. What is the probability a randomly selected airport will have a summer average temperature between 62 and 72 degrees F? **d.** 0.68

e. SeaTac's airport has a summer average temperature of 68 degrees. Calculate the z-score.

$$z = \frac{68-67}{5} = \frac{1}{5} = 0.2$$

f. LAX has a z-score of 2.6. What is the airport's summer average temperature?

$$2.6 = \frac{x-67}{5} \qquad x = 80°F$$

g. A sample of 11 western regional airports reported their annual average temperatures. Calculate the margin of error for this sample. Round your answer to the nearest degree.

$$MOE = \pm \frac{2\sigma}{\sqrt{n}} = \pm \frac{2(5)}{\sqrt{11}} = \pm 3.015 = \boxed{\pm 3°F}$$

h. The airport in Anchorage, Alaska claims their average airport temperature in the summer is 57 degrees. Do you agree with their claim? Use a range of reasonable means to support your answer.

Range of Reasonable Means: $67° \pm 3°F = 64° - 70°$ ∴ No, 57° is below the Range of Reasonable Means.

12. What happens to the margin of error when the sample size decreases and all other factors remain the same? Explain your reasoning. $MOE = \pm \frac{2\sigma}{\sqrt{n}}$ as n decreases, MOE increases.

Name Key

Topic 5 Midterm Review

1. Simplify each radical expression. using absolute value bars when necessary. Write your answer in reduced radical form.

a. $-3\sqrt{12}+3\sqrt{3}+3\sqrt{20}$

$-3\sqrt{4}\cdot\sqrt{3}+3\sqrt{3}+3\sqrt{4}\cdot\sqrt{5}$

$-6\sqrt{3}+3\sqrt{3}+6\sqrt{5}$

$=-3\sqrt{3}+6\sqrt{5}$

b. $-2\sqrt[3]{24}+\sqrt[3]{162}$

$-2\sqrt[3]{3}\cdot\sqrt[3]{3}+\sqrt[3]{27}\cdot\sqrt[3]{6}$

$=-4\sqrt[3]{3}+3\sqrt[3]{6}$

c. $(\sqrt{2a}-5)(7\sqrt{2a}-5)$

$\sqrt{2a}\cdot 7\sqrt{2a}-5\sqrt{2a}-35\sqrt{2a}+25$

$=14|a|-40\sqrt{2a}+25$

d. $\sqrt[3]{-16x^3b^8}$

$=\sqrt[3]{-8\cdot 2x^3b^8}$

$=-2\sqrt[3]{2x^3b^6b^2}$

$=-2xb^2\sqrt[3]{2b^2}$

e. $\sqrt{48x^2y^6}$

$=\sqrt{16\cdot 3x^2y^6}$

$=4|xy^3|\sqrt{3}$

f. $\sqrt[3]{\frac{12x}{2y^2}}$

$=\frac{\sqrt[3]{12x}}{\sqrt[3]{2y^2}}\cdot\frac{\sqrt[3]{4y}}{\sqrt[3]{4y}}$

$=\frac{\sqrt[3]{48xy}}{2y}=\frac{2\sqrt[3]{6xy}}{2y}$

$=\frac{\sqrt[3]{6xy}}{y}$

g. $\frac{3x}{4-\sqrt{2}}\left(\frac{4+\sqrt{2}}{4+\sqrt{2}}\right)$

$=\frac{3x(4+\sqrt{2})}{16-2}=\frac{12x+3x\sqrt{2}}{14}$

2. Rewrite each expression using the properties of exponents. Simplify as much as possible and write your final answer with rational exponents with no negative exponents.

a. $\frac{2h^3j^{-3}k^4}{3jk}$

$\frac{2h^3k^3}{3j^4}$

b. $\left(\frac{27x^{-3}y^6}{16zx^6}\right)^{\frac{2}{3}}$

$\frac{27^{2/3}x^{-2}y^4}{16^{2/3}z^{2/3}x^4}$

$=\frac{9y^4}{16^{2/3}x^6z^{2/3}}$

c. $\frac{a^3(b^2c)^{-1}}{(a^{-3}b)^2c^{-2}}$

$=\frac{a^3b^{-8}c^{-4}}{a^{-6}b^2c^{-2}}$

$=\frac{a^9}{b^{10}c^2}$

3. Solve each equation.

a. $4+\sqrt[3]{x-6}=2$

$\sqrt[3]{x-6}=-2$

$x-6=(-2)^3$

$x-6=-8$

$x=-2$ ✓

b. $\sqrt{19-3x}-1=3x$

$\sqrt{19-3x}=3x+1$

$19-3x=(3x+1)(3x+1)$

$19-3x=9x^2+6x+1$

$0=9x^2+9x-18$

$0=9(x+2)(x-1)$

$x=-2$ ✗ extraneous! $x=1$ ✓

c. $(x+2)^{\frac{2}{3}}=9$ 3/2

$x+2=9$

$x+2=(\sqrt[3]{9})^3$

$x+2=(\pm 3)^3$

$x+2=27$ $x+2=-27$

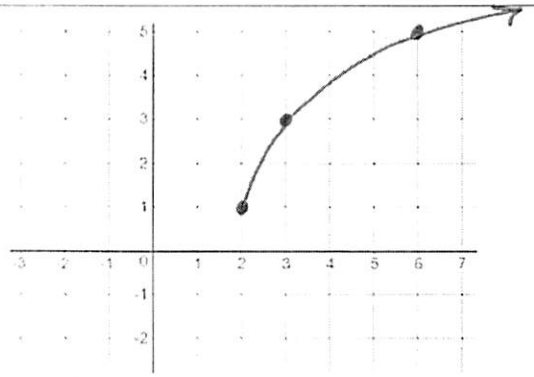
✓ $x=25$ ✓ $x=-29$

4. For the equation $g(x) = -4\sqrt[3]{-(x-5)} + 5$, select all the transformations that apply.

[A] Vertical Dilation by a factor of 4
[B] Reflection over the x-axis
[C] Translated to the left 5
[D] Translated up 5
[E] Horizontal Dilation by a factor of 4
[F] Reflection over the y-axis
[G] Translated to the right 5
[H] Translated down 5

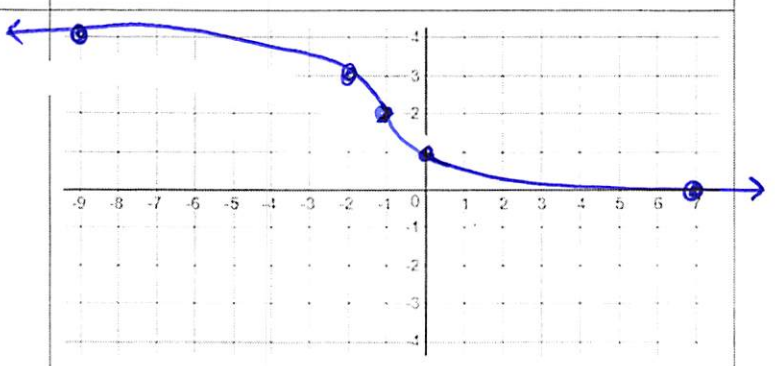
5. For each equation, graph, and fill out the features asked.

a. $f(x) = 2\sqrt{x-2} + 1$



Domain: $x \in [2, \infty)$	Range: $y \in [1, \infty)$
Interval where $f(x)$ is increasing:	
Average rate of change over the interval $x \in [3, 6]$: $2/3$	

b. $g(x) = -\sqrt[3]{x+1} + 2$



Domain: $x \in \mathbb{R}$	Range: $y \in \mathbb{R}$
Point of inflection: $(-1, 2)$	
Interval where $g(x) > 0$: $x \in (-\infty, 7)$	

For 6-7: Write the rules for the given functions, simplify as much as possible, and state its domain.

6. $g(x) = 2x^2 + 11x + 5$ and $f(x) = x + 5$ ($\mathbb{R}$ $\mathbb{R}$)

a. $(g - f)(x)$

$= 2x^2 + 11x + 5 - (x+5)$

$= 2x^2 + 10x$

Domain: $x \in \mathbb{R}$

b. $\left(\frac{f}{g}\right)(x) = \frac{x+5}{2x^2+11x+5}$

$= \frac{x+5}{(x+5)(2x+1)} = \frac{1}{2x+1}; x \neq -5$

Domain: $x \in \mathbb{R}; x \neq -5, -\frac{1}{2}$

c. $(f \circ g)(x) = f(g(x))$

$= (2x^2+11x+5) + 5$

$= 2x^2 + 11x + 10$

Domain: $x \in \mathbb{R}$

7. $j(x) = \sqrt{x+3}$ and $k(x) = \frac{1}{2x}$

a. $(j \cdot k)(x)$

$\sqrt{x+3} \cdot \frac{1}{2x} = \frac{\sqrt{x+3}}{2x}$

Domain: $x \geq -3$, $x \neq 0$

b. $(k \circ j)(x) = k(j(x))$

$\frac{1}{2(\sqrt{x+3})} = \frac{1}{2\sqrt{x+3}}$

Domain: $x > -3$

c. $(j \circ k)(x) = j(k(x))$

$= \sqrt{\frac{1}{2x} + 3}$

Domain: $x \in \mathbb{R}; x \geq -\frac{1}{6}$, $x \neq 0$

Multiple Choice: For 7-8 Identify the inverse of each function.

7. $g(x) = 2x^3 + 3$

[A] $g^{-1}(x) = \left(\frac{x-3}{2}\right)^3$

[B] $g^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

[C] $g^{-1}(x) = (2(x+3))^3$

[D] $g^{-1}(x) = \sqrt[3]{2(x+3)}$

8. $g(x) = \frac{-4+5x}{3}$

[A] $g^{-1}(x) = 3(x+4)$

[B] $g^{-1}(x) = \frac{-5x+4}{3}$

[C] $g^{-1}(x) = \frac{3x+4}{5}$

[D] $g^{-1}(x) = 5(x-4)$

9. Which of the following must be true about $f(x)$ and $f^{-1}(x)$? *Select all that apply.*

[A] If the domain of $f(x)$ is all real numbers, then the domain of $f^{-1}(x)$ is all real numbers.

[B] The graphs of $f(x)$ and $f^{-1}(x)$ are reflections over the line $y = x$.

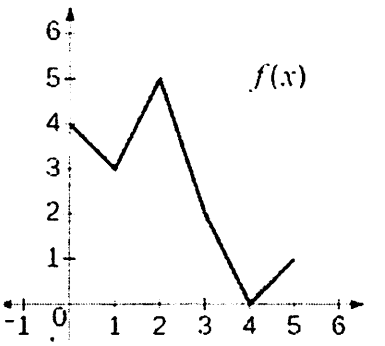
[C] $f(f^{-1}(5)) = f^{-1}(f(5))$.

[D] $f^{-1}(x)$ does not have to pass the vertical line test in order to be a function.

[E] $f(x)$ passes the vertical line test.

10. Use the table of $h(x)$, graph of $f(x)$ and equation $g(x) = \frac{1}{2}\sqrt{x+4}$ to evaluate each of the following.

x	$h(x)$
-1	-3
0	4
1	11
2	18



a. $f(g(0)) = f(1) = 3$

b. $h(f(3)) = h(2) = 18$

c. $g(f(g(12))) = g(f(2)) = g(5) = 1.5$

d. x, when $g(f(x)) = 1.5$

$1.5 = \frac{1}{2}\sqrt{x+4}$

$3 = \sqrt{x+4}$

$9 = x+4$

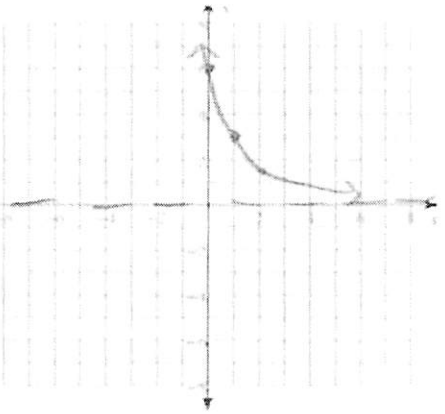
$f(x) = 5$

$x = 2$

Name Key Date _____ Period _____

Algebra 2: Topic 6 Midterm Review

1. Sketch the graph of $h(x)=3\cdot 2^{-(x-1)}$ on the axes at right.



2. **Multiple Response**: Which of the following are true for $h(x)=3\cdot 2^{-(x-1)}$. *Select all that apply.*

[A] x- interval where $h(x)>0$: $x\in\mathbb{R}$

[B] x- interval where $h(x)>0$: Does Not Exist

[C] $h(x)$ average rate of change for $x\in[-2,1]$ is -7.

[D] $h(x)$ average rate of change for $x\in[-2,1]$ is $-\frac{1}{7}$

[E] y-intercept: $(0,6)$

[F] x-intercept: $(6,0)$

[G] Asymptote: $x=0$

[H] Asymptote: $y=0$

3. Describe the end behavior for $h(x)=3\cdot 2^{-(x-1)}$. *Select all that apply.*

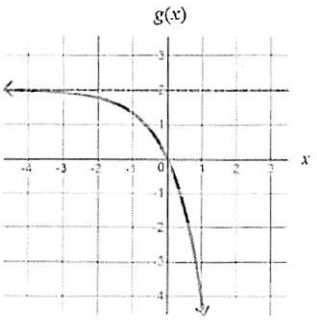
[A] As $x\to\infty$, $h(x)\to 0^+$

[B] As $x\to\infty$, $h(x)\to 0^-$

[C] As $x\to-\infty$, $h(x)\to\infty$

[D] As $x\to-\infty$, $h(x)\to-\infty$

Questions 4 & 5 refer to $g(x)$, the function graphed at right.



4. **Multiple Choice**: Select the equation for $g(x)$.

[A] $g(x)=-2\cdot 3^{x+1}$

[B] $g(x)=-2\cdot 3^{x}+2$

[C] $g(x)=2\cdot 3^{-(x+1)}$

[D] $g(x)=2\cdot 3^{-x}+2$

5. **Multiple Response**: Select all the transformations on the function $g(x)$ from the parent function $f(x)=3^x$.

[A] Reflection over the x-axis

[B] Reflected over the y-axis

[C] Vertical dilation scale factor of 3

[D] Vertical dilation scale factor of 2

[E] Vertical translation up 2

[F] Horizontal translation left 1

6.

a. $4^{3(x-4)}=64^{5x}$

$4^{3(x-4)}=4^{3(5x)}$

$3(x-4)=3(5x)$

$x-4=5x$

$-4=4x$

$x=-1$

b. $27=\left(\frac{1}{9}\right)^{-4x}$

$3^3=(3^{-2})^{-4x}$

$\frac{3}{3}=\frac{8x}{3}$

$x=8/3$

c. $5\log(x+3)=25$

$\frac{5}{5}\quad\frac{5}{5}$

$\log(x+3)=5$

$10^5=x+3$

$100000=x+3$

$-3\quad -3$

$x=99997$

d. $\log_3 81=\log_4(5x+6)$

$4=\log_4(5x+6)$

$4^4=5x+6$

$256=5x+6$

$-6\quad -6$

$\frac{250}{5}=\frac{5x}{5}$

$x=50$

e. $\log_x 64=-3$

$x^{-3}=64$

$\left(\frac{1}{4}\right)^{-3}=64$

$x=1/4$

f. $\log_9\frac{1}{81}=x$

$9^x=\frac{1}{81}$

$x=-2$

g. $3^{x-7}=\frac{1}{81}$

$3^{x-7}=3^{-4}$

$x-7=-4$

$+7\quad +7$

$x=3$

h. $\log_{\frac{1}{32}}x=\frac{3}{5}$

$32^{3/5}=x$

$(\sqrt[5]{32})^3=x$

$2^3=x$

$8=x$

7. You deposit \$6,500 into an account paying 8% annual interest compounded monthly.

a. Write an equation to represent the amount of money, A, in your account after t years.

$A(t) = 6{,}500\left(1 + \frac{.08}{12}\right)^{12t}$

b. Find the amount of money in the account be after 4 years.

$A(4) = 6{,}500\left(1 + \frac{.08}{12}\right)^{12 \cdot 4}$ $\quad A(4) = 8941.82965$ $\quad \$\ 8941.830$

c. **Multiple Choice**: After how many years will the account be worth \$11358.24 (Remember: do not round until you write your final solution).

[A] 6 years [B] 7 years [C] 84 years [D] 83 years

graph $y = 11358.24$ and $A(t) = 6500\left(1 + \frac{.08}{12}\right)^{12t}$ intersection!

8. Field mice started making an abandoned barn their home. After two months there was only 25 mice. The population continued to grow exponentially and after 14 months the barn was home to 9,808 mice.

a. Write an equation $M(t)$ that represents the mice population after t amount of months.

$(2, 25)\ (14, 9808)$

$25 = ab^2 \quad 9808 = ab^{14}$

$9808 = \left(\frac{25}{b^2}\right)b^{14}$

$\frac{9808}{25} = b^{12}$

$\therefore b = 1.64488\ldots$

$25 = a(1.6448\ldots)^2 \therefore a = 9.23988\ldots$

$M(t) = 9.240(1.645)^t$

b. How many whole mice will there be after 2 years?

$M(24) = 9.24(1.644\ldots)^{24} = 1422156.85$ 1422156 mice

c. **Multiple Choice**: What is the growth rate of the mice?

[A] 0.645% [B] 1.645% [C] 35.511% [D] 64.489%

9. Create your own decay scenario. Then write an equation to represent your scenario.

answers vary ex. of eqn.: $g(t) = 100(.5)^t$

$0 < b < 1$ = decay

10. **Multiple Response**: Which of the following must true? *Select all the apply.*

[A] Assuming no transformations, the x-intercept of $\log_b x$ is $(b, 0)$.

[B] Assuming no transformations, the x-intercept of $\log_b x$ is $(1, 0)$.

[C] If $f(x) = 2^x$, then $f^{-1}(x) = \log_2 x$.

[D] If $f(x) = \log x$, then $f^{-1}(x) = x^{10}$.

[E] All exponential functions of the form $y = a \cdot b^{(x-h)} + k$ have a horizontal asymptote of $y = k$.

[F] All logarithmic functions of the form $y = a\log_b(x-h) + k$ have a vertical asymptote of $x = k$.

11. **Multiple Choice**: Find the inverse of $g(x) = -5\log_3(x-4) + 2$

[A] $g^{-1}(x) = -5 \cdot 3^x + 4$ [B] $g^{-1}(x) = -5 \cdot 3^{x-2} + 4$ [C] $g^{-1}(x) = 3^{\frac{-(x-2)}{5}} + 4$

$X = -5\log_3(g(x) - 4) + 2$ $\quad \frac{X-2}{-5} = \log_3(g(x) - 4)$

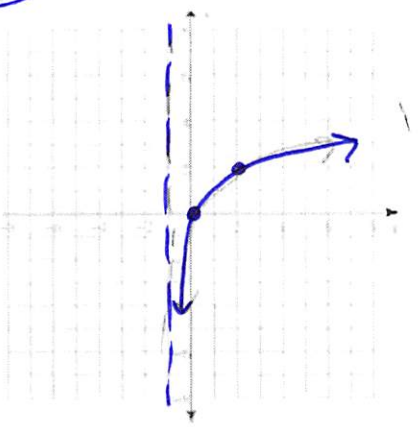
$3^{\frac{X-2}{-5}} = g(x) - 4 \quad \therefore g^{-1}(x) = 3^{\frac{X-2}{-5}} + 4$

$\log_2(h(x)+1) = x+13$

12. Multiple Choice: Convert $h(x) = 2^{x+13} - 1$ to logarithmic form.

[A] $h(x) = \log_2(x-1)+13$ [B] $h(x) = \log_2(x+13)-1$ [C] $h(x) = \log_2(x+1)-13$

13. Graph the following function $g(x) = 2\log_3(x+1)$ and find the following:



Domain: $x \in (-1, \infty)$

Range: $g(x) \in \mathbb{R}$

x-intercept: $(0, 0)$

Asymptote: $x = -1$

End Behavior:

$x \to \infty, g(x) \to \infty$

$x \to -1^+, g(x) \to -\infty$

14. Write each series using sigma notation, then find the sum. If the sum doesn't exist explain.

a. $\frac{1}{2} + \frac{1}{10} + \frac{1}{50} \ldots$

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)^{n-1}$

$\frac{\frac{1}{2}}{1-\frac{1}{5}} = 5/8 = 0.625$

b. $-6 + -12 + -24 + \ldots + -384$

$\sum_{n=1}^{7} -6(2)^{n-1} = -762$

$S_7 = \frac{-6(1-2^7)}{1-2} =$

$-384 = -6(2)^{n-1}$

$64 = 2^{n-1}$

$2^6 = 2^{n-1}$

$\therefore n = 7$

For Questions 15 & 16, use the explicit definition: $a_n = \begin{cases} 2, n=1 \\ 3a_{n-1}, n>1 \end{cases}$

15. Multiple Choice: Find the explicit definition given the recursive definition above.

[A] $a_n = 3 \cdot 2^n; n \geq 1$ [B] $a_n = 2 \cdot 3^n; n \geq 1$ [C] $a_n = 3 \cdot 2^{n-1}; n \geq 1$ [D] $a_n = 2 \cdot 3^{n-1}; n \geq 1$

16. Multiple Choice: Find the value of a_4.

[A] $a_4 = 162$ [B] $a_4 = 54$ [C] $a_4 = 216$ [D] $a_4 = 1296$

17. Aditya sent a chain letter to 100 of his friends and asked them to forward it to their friends. Each day since then, the number of people who received the letter was 20% more than the day before.

a. Let g_n represent the number of people who received the letter on the n^{th} day since Aditya sent it. Write an explicit definition to represent the scenario.

$g_n = 100 \cdot 1.2^{n-1}; n \geq 1$

b. On the 10th day, how many people received the letter THAT day?

$g_{10} = 100 \cdot 1.2^{10-1} = 515.978\ldots$ 515 people

c. Find number of days, to the nearest whole number, when 358 people had received the letter in TOTAL.

$358 = 1.2^{n-1}$

$\log_{1.2} 358 = n-1$

$n = 7.99\ldots$

after 8 days