5.4 Notes: Solving Radical Functions

① $\sqrt{x+5} + 1 = 3$

$(\sqrt{x+5})^2 = 4^2$ Isolate the $\sqrt{}$

$x + 5 = 16$

$x = 11$

Check your answers

$\sqrt{11+5} - 1 = 3$

$\sqrt{16} - 1 = 3$

$4 - 1 = 3$

$3 = 3$ ✓

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② $(\sqrt{3x-2})^2 = (x-4)^2$ $\sqrt{3x-2} = x - 4$

$3x - 2 = (x-4)(x-4)$

$3x - 2 = x^2 - 8x + 16$

$0 = x^2 - 11x + 18$ Factor

$0 = (x-9)(x-2)$

$x = 9, \quad x = 2$ ← Extraneous Solution

Check: $\sqrt{3(9)-2} = 9 - 4$

$\sqrt{27-2} = 5$

$\sqrt{25} = 5$

$5 = 5$ ✓

$\sqrt{(3)(2)-2} = 2 - 4$

$\sqrt{6-2} = -2$

$\sqrt{4} = -2$

$2 \neq -2$

③ $x = \sqrt{7x+8}$

$x^2 = \left(\sqrt{7x+8}\right)^2$ Sq both sides

$x^2 = 7x + 8$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = 8, \quad x = -1$

Check

$8 = \sqrt{7(8)+8}$

$8 = \sqrt{56+8}$

$8 = \sqrt{64}$

$8 = 8$ ✓

$-1 = \sqrt{7(-1)+8}$

$-1 = \sqrt{-7+8}$

$-1 = \sqrt{1}$

$-1 \neq 1$ ✗

↑ Extraneous

only $x = 8$ is a solution

④ $x+2^{\,2} = \sqrt{x+2}^{\,2}$

$(x+2)^2 = x+2$

$x^2 + 4x + 4 = x + 2$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2, \quad x = -1$

Both solutions work

Solving rational equations

① $(x^2+5x+5)^{5/2} = 1$

power ↙ root ↙

$((x^2+5x+5)^{5/2})^{2/5} = 1^{2/5}$ Undo the exponent by taking to reciprocal power

$x^2+5x+5 = 1$

$x^2+5x+4 = 0$

$(x+4)(x+1) = 0$

$x = -4,\ x = -1$

<u>check</u>

$((-4)^2+5(-4)+5)^{5/2} = 1$

$(16-20+5)^{5/2} = 1$

$(1)^{5/2} = 1$

$1 = 1$ ✓

$((-1)^2+5(-1)+5)^{5/2} = 1$

$(1-5+5)^{5/2} = 1$

$(1)^{5/2} = 1$

$1 = 1$ ✓

Both $x = -4$ & $x = -1$ are sol.

(2) $(x+18)^{3/2} = (x-2)^3$

$\left((x+18)^{3/2}\right)^{2/3} = \left((x-2)^3\right)^{2/3}$ Take to reciprocal power

$x+18 = (x-2)^2$

$x+18 = (x-2)(x-2)$

$x+18 = x^2-4x+4$

$0 = x^2-5x-14$

$0 = (x-7)(x+2)$

$x = 7, -2$

Check

$(7+18)^{3/2} = (7-2)^3$

$(25)^{3/2} = 5^3$

$125 = 125$ ✓

$(-2+18)^{3/2} = (-2-2)^3$

$(16)^{3/2} = (-4)^3$

$64 \neq -64$

only $x = 7$ is a sol.

(3) $0.5(x^2+5x+136)^{2/3} = 50$

$\left((x^2+5x+136)^{2/3}\right)^{3/2} = (100)^{3/2}$

$x^2+5x+136 = 1000$

$x^2+5x-864 = 0$

$(x+32)(x-27) = 0$

$\boxed{x = -32, 27}$